Amsterdam, Montague '97

Intensional Logic and Two-sorted Type Theory

by

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1. Notation

Types

2T: (*two-sorted types*):

$2T \to s$; $2T \to e$; $2T \to t$; $2T \to <2T,2T>$.

IT (*intensional types/IL-types*):

$IT \to e$; $IT \to t$; $IT \to <IT,IT>$; $IT \to <s,IT>$.

IT^+: $IT \cup \{s\}$.

Languages

$(Ty2_\tau)_{\tau\in 2T}$ (*two-sorted type theory*):

$(Var_\tau)_{\tau\in 2T}$ and $(Con_\tau)_{\tau\in 2T}$ [infinite for each τ] plus:

functional application, λ-abstraction, and identity.

$(IL^+_\tau)_{\tau\in IT^+}$ (*intensional logic plus world variables*):

$(Var_\tau)_{\tau\in IT^+}$ and $(Con_{<s,\tau>})_{\tau\in IT^+}$ [infinite for each τ] plus:

functional application, λ-abstraction, and identity [all three restricted to IT^+].

$(IL^*_\tau)_{\tau\in IT^+}$ (*Gallin's IL*):

like IL^+, except:

- no IL^*_s ($s \notin$ IT!)

- no type s variables $\neq r$ ($v_{0,s}$).

IL (*Montague's intensional logic*):

[esssentially] notational variant of IL^*.

$(\mathrm{Ty2}^-_\tau)_{\tau \in IT^+}$ (*the relevant fragment*)

Ty2 with IL$^+$-parameters (free variables and constants).

2-functors:

$(\mathrm{Ty2}_\tau)_{\tau \in 2T \setminus IT^+}$.

Ontology and semantics

standard; notation:

$(D_\tau)_{\tau \in 2T}$ for the ontology, and

$|\alpha|^{M,g}$ for α's extension according to the model M and the assignment g.

2. From Ty2$^-$ to IL$^+$

Theorem I: For any $\alpha \in$ Ty2$^-$ there is an equivalent $\alpha^+ \in$ IL$^+$.

Proposition I: If $\alpha \in$ Ty2$^-$ is λ-reduced and β is a 2-functor occurring in α, then either:

(a) β contains a free variable alien to IL$^+$, or:

(b) β is of the form $(\lambda x\ \gamma)$.

Corollary: 2-funcors occurring in λ-reduced Ty2$^-$-expressions are either parts of larger 2-functors or immediate parts of equations.

Consequence (for the proof of Theorem I): coding will do.

Sketch of proof:

Step 1: Define a type correspondence $^+$: $2T \rightarrow IT^+$ by:

$\tau^+ = \tau$, if τ is basic;

$(\tau s)^+ = (\tau^+(st))$;

$(\sigma\tau)^+ = (\sigma^+\tau^+)$, if $\tau \neq s$.

Step 2: Define another function $^+$ [sorry!] mapping any object of type $2 \in 2T$ on a *set* of τ^+-objects s.th.:

$U^+ = \emptyset$, for any U;

$\tau = \tau^+ \Rightarrow U^+ = \{U\}$;

$U \neq V \Rightarrow U^+ \cap V^+ = \emptyset$.

Step 3: Define a 3rd $^+$ from $Ty2_\tau^-$ to $IL^+_{\tau^+ r}$.

Step 4: Show that the 3rd $^+$ partially characterizes the second one:

$\emptyset \neq |\alpha^+| \subseteq |\alpha|^+$.

Step 5: Use Proposition I to prove Theorem I.

3. From IL$^+$ to IL*

Theorem II: If $\alpha \in IL_\tau^+$, $\tau \neq s$ and r is the only free variable of type s in α, then there exists an equivalent $\alpha^- \in IL^*_\tau$.

Lemma II: If $\tau \in IT$ and $\alpha \in IL^+_\tau$, then the following conditions hold:

(a) α is equivalent to $\alpha_0(\alpha_1)$, and they share the same parameters.

(b) If $x \in Var_s$ occurs in α_0, then x is r.

(c) If $x \in Var_s$ occurs in α_1, then x occurs freely in α_1 and x is not r.

(1) (a) $R(x)$

I R $[\lambda f\ f(\cancel{r})]\quad (\lambda S\ S(x))$

(b) $R(x)(r)$

I R $[\lambda g\ g(\lambda f\ f(\cancel{r}))(r)]\quad (\lambda G\ G(\lambda S\ S(x)))$

(c) $R(x)(r)(y)$

$[\lambda h\ h(\lambda g\ g(\lambda f\ f(\cancel{r}))(r))]\quad (\lambda H\ H(\lambda G\ G(\lambda S\ S(x)))(y))$

I R

(2) $\lambda r\ \beta$

$[\lambda X\ \lambda r\ X(\beta_0)]\quad (\lambda Y\ Y(\beta_1))$

(3) If $x \in Var_s \setminus \{r\}$, then α_0 is:

$$(\lambda f\ (\lambda g\ (\lambda r\ \ g\ (f\ (\beta_1\ [x/r\ ]_0)\ ))\ (\beta_0))$$

and α_1 is:

$$\lambda h\ h\ (\beta_1\ [x/r\ ]_1)\ ,$$

where '$[x/r]$' denotes the replacement of all free occurrences of x by r, f is of type $((\sigma(\beta_1[x/r)\sigma(\beta))(\sigma(\beta))$ $(=:\sigma(\alpha))$, and g and h are new variables of types $(\sigma(\beta)\tau'')$, and $(\sigma(\beta_1[x/r])\sigma(\beta))$, respectively.

4. Conclusions

<u>Theorem III</u>: If $\alpha \in Ty2^-$, $\alpha \neq r$ and $FV_s(\alpha) = \{r\}$, then α is equivalent to an IL^*-expression.

References

Gallin, Daniel: ***Intensional and Higher-order Modal Logic.*** Amsterdam 1975.

Montague, Richard : Universal grammar. ***Theoria*** 36 (1970), 373-98.