Worlds and Models

1. Principles of Truth-conditional Semantics

Most Certain Principle Cresswell (1982: 69)
If S_1 and S_2 are declarative sentences such that, under given circumstances, S_1 is true whereas S_2 is not, then S_1 and S_2 differ in meaning.

Basic Principle of Truth-Conditional Semantics Wittgenstein (1922: 4.431)
Any two declarative sentences agree in meaning just in case they agree in their truth conditions.

(1) **John loves Mary, who does not like John.**
(2) **Mary loves John, who does not like Mary.**
(3) **Someone is buying a car.** Abbott & Hauser (1995: 6, n. 10)
(4) **Someone is selling a car.**
(5) **Mary is buying a car.**
(6) **Mary is selling a car.**

Substitution Principle Frege (1892)
If two non-sentential expressions have the same meaning, either may replace the other in all positions within any sentence without thereby affecting the truth conditions of that sentence.

Principle of Compositionality Tarski (1936); Montague (1970)
The meaning of a complex expression functionally depends on the meanings of its immediate parts and the way in which they are combined.

Frege's Context Principle [Urform] Frege (1884: 71)
It suffices for a sentence as a whole to have a sense; thereby its parts, too, receive contents.

Rather Plausible Principle Frege (1892)
If T_1 and T_2 are terms such that, under given circumstances, T_1 refers to something to which T_2 does not refer, then T_1 and T_2 differ in meaning.

Extended Basic Principle of Truth-Conditional Semantics [EBP]
Any two declarative sentences agree in meaning just in case they agree in their truth conditions; and any two terms agree in meaning just in case they agree in their reference conditions.

Frege's Later Context Principle Frege (1891; 1892)
If not specified by the *EBP*, the meaning of an expression A is the contribution A makes to the meanings of (certain) expressions B in which A (immediately) occurs, i.e. a function assigning meanings of host expressions (in which A occurs) to meanings of positions (in which A occurs).

Digression: Inscrutability of Reference Quine (1960: 50ff.);Williams (2005)
(7) **Here comes Peter Cottontail.**

2. Possible Worlds Semantics

Carnap (1947)

2.1 Logical Space

Wittgenstein (1922); Lewis (1986)

> *Extensional Version of Frege's Context Principle* Frege (1892)
> If not specified by the *EBP*, the extension of an expression ***A*** is the contribution ***A*** makes to the extensions of (certain) expressions in which ***A*** (immediately) occurs, i.e. a function assigning extensions of host expressions (in which ***A*** occurs) to extensions of positions (in which ***A*** occurs).

<u>Vastness</u>: *Circumstances may be as hypothetical as can be.*
<u>Detail</u>: *Circumstances are as specific as can be.*

(8) **The physicist who discovered the X-rays died in Munich.**
(9) **The first Nobel laureate in physics died in Munich.**
(10) **Dr Who ate himself .**
(11) **Dr Who is his own father.**
(12) **General Beauregard Lee is a woodchuck.**
(13) **General Beauregard Lee is a whistlepig.**
(14) **General Beauregard Lee is a whistlepig, and if he is a whistlepig, he is a woodchuck.**
(15) **General Beauregard Lee is a whistlepig, and if he is a woodchuck, he is a woodchuck.**
(16) **General Beauregard Lee is a whistlepig, and either he lives in Georgia, or he does not live in Georgia.**
(17) $\| \boldsymbol{P} \|^w (\| \boldsymbol{T} \|^w) = \| \boldsymbol{TP} \|^w$
(18) **Every boy fancies Mary and Jane sulks.**
[18] [[[**Every boy**] [**fancies Mary**]] **and** [**Jane sulks**]]

(19a) $\| \textbf{every} \|^w = \lambda P.\ \lambda Q.\ \vdash P \subseteq Q \dashv$
(b) $\| \textbf{boy} \|^w = \lambda x.\ \vdash x \text{ is a boy in } w \dashv$
(c) $\| \textbf{fancies} \|^w = \lambda x.\ \lambda y.\ \vdash y \text{ fancies } x \text{ in } w \dashv$
(d) $\| \textbf{Mary} \|^w = \text{Mary}$
(e) $\| \textbf{and} \|^w = \lambda u.\ \lambda v.\ u \times v$
(f) $\| \textbf{Jane} \|^w = \text{Jane}$
(g) $\| \textbf{sulks} \|^w = \lambda x.\ \vdash x \text{ sulks in } w \dashv$

- '$\lambda x.$ ...x...' denotes the function assigning to x whatever '...x...' denotes. In particular:
  $[\lambda x.\ \ldots x \ldots](a) = \ldots a \ldots$ [= β-*conversion*, or λ-*conversion*]
- '$\vdash \ldots \dashv$' is short for 'the truth value that is identical to 1 just in case ...'. In particular:
  $\lambda x.\ \vdash \ldots \dashv$ characterises $\{x \mid \ldots x \ldots\}$

(20a) $\| \textbf{every boy} \|^w = [\lambda Q.\ \vdash [\lambda x.\ x \text{ is a boy in } w] \subseteq Q \dashv]$
(b) $\| \textbf{fancies Mary} \|^w = [\lambda x.\ \vdash y \text{ fancies Mary in } w \dashv]$
(c) $\| \textbf{Jane sulks} \|^w = \ \vdash \text{Jane sulks in } w \dashv$

(21a) $\|\,\textbf{every boy fancies Mary}\,\|^{w}$

= $\vdash \{x.\ \vdash x \text{ is a boy in } w\,\} \subseteq \{y \mid y \text{ fancies Mary in } w\,\} \dashv$

(b) $\|\,\textbf{every boy fancies Mary and Jane sulks}\,\|^{w}$

= $\vdash \text{Jane sulks in } w \dashv \times \vdash \{x \mid x \text{ is a boy in } w\} \subseteq \{y \mid y \text{ fancies Mary in } w\} \dashv$

(22a) If ***D N*** is a quantifier phrase, where ***D*** is a quantificational determiner and ***N*** is a count noun, then:

$\|\boldsymbol{D\ N}\|^{w} = \|\boldsymbol{D}\|^{w}(\|\boldsymbol{N}\|^{w})$ [cf. (20a)]

(b) If ***V T*** is a predicate, where ***V*** is a transitive verb and ***T*** is a term, then:

$\|\boldsymbol{V\ T}\|^{w} = \|\boldsymbol{V}\|^{w}(\|\boldsymbol{T}\|^{w})$ [cf. (20b)]

(c) If ***T P*** is a declarative sentence, where ***T*** is a term and ***P*** is a predicate, then:

$\|\boldsymbol{T\ P}\|^{w} = \|\boldsymbol{P}\|^{w}(\|\boldsymbol{T}\|^{w})$ [cf. (20c)]

(d) If ***Q P*** is a declarative sentence, where ***Q*** is a quantifier phrase and ***P*** is a predicate, then:

$\|\boldsymbol{Q\ P}\|^{w} = \|\boldsymbol{Q}\|^{w}(\|\boldsymbol{P}\|^{w})$ [cf. (21a)]

(e) If ***A C B*** is a declarative sentence, where ***C*** is a coordinating conjunction and ***A*** and ***B*** are declarative sentences, then:

$\|\boldsymbol{A\ C\ B}\|^{w} = \|\boldsymbol{C}\|^{w}(\|\boldsymbol{B}\|^{w})(\|\boldsymbol{A}\|^{w})$ [cf. (21b)]

2.2 Material Models

(23) $\|\boldsymbol{T\ P}\|^{w} = \|\boldsymbol{P}\|^{w}(\|\boldsymbol{T}\|^{w})$ [= (22c)]

(24) $\|\boldsymbol{V\ Q}\|^{w} = \lambda x.\ \|\boldsymbol{Q}\|^{w}(\lambda y.\ \|\boldsymbol{V}\|^{w}(y)(x))$

(25) $\|\boldsymbol{A}\|^{w} = F_{w}(\boldsymbol{A})$

(26a) $\|\textbf{and}\|^{w} = \lambda u.\ \lambda v.\ u \times v$ [= (19e)]

(b) $\|\textbf{or}\|^{w} = \lambda u.\ \lambda v.\ u + v - u \times v$

(c) $\|\textbf{not}\|^{w} = \lambda u.\ 1 - u$

(27a) $\|\textbf{every}\|^{w} = \lambda P.\ \lambda \mathrm{Q}.\ \vdash P \subseteq Q \dashv$ [= (19a)]

(b) $\|\textbf{no}\|^{w} = \lambda P.\ \lambda \mathrm{Q}.\ \vdash P \cap Q = \varnothing \dashv$

(c) $\|\textbf{is}\|^{w} = \lambda x.\ \lambda y.\ \vdash x = y \dashv$

(28a) $\|\textbf{boy}\|^{w} = \lambda x.\ \vdash x \text{ is a boy in } w \dashv$ [= (19b)]

(b) $\|\textbf{fancies}\|^{w} = \lambda x.\ \lambda y.\ \vdash y \text{ fancies } x \text{ in } w \dashv$ [= (19c)]

(c) $\|\textbf{sulks}\|^{w} = \lambda x.\ \vdash x \text{ sulks in } w \dashv$ [= (19g)]

Definition

Given a possible world w and a language L, the *material model* (*for L based on w*) is the pair $\mathcal{M}_w = (U_w, F_w)$ consisting of the *domain* of individuals U_w in w and the lexical interpretation function F_w which assigns to every non-logical lexical expression $\boldsymbol{A}$ of L the extension of $\boldsymbol{A}$ at w.

(29) For any expression $\boldsymbol{A}$ of E and any material model $\mathcal{M}_w = (U_w, F_w)$ for E, the *extension of* $\boldsymbol{A}$ *relative to* $\mathcal{M}_w$ – $|\,\boldsymbol{A}\,|^{\mathcal{M}_w}$ – is determined by the following induction (on the grammatical complexity of $\boldsymbol{A}$):

(i-a) $|\,\mathbf{and}\,|^{\mathcal{M}_w} = \lambda u.\ \lambda v.\ u \times v$ … where $u \in \{0,1\}$ and $v \in \{0,1\}$

(i-b) $|\,\mathbf{or}\,|^{\mathcal{M}_w} = \lambda u.\ \lambda v.\ u + v - u \times v$ … where $u \in \{0,1\}$ and $v \in \{0,1$

… …

(ii-a) $|\,\mathbf{every}\,|^{\mathcal{M}_w} = \lambda P.\ \lambda Q.\ \vdash P \subseteq Q \dashv$ where $P \subseteq U_w$ and $Q \subseteq U_w$

(ii-b) $\|\,\mathbf{no}\,\|^{w} = \lambda P.\ \lambda Q.\ \vdash P \cap Q = \emptyset \dashv$ where $P \subseteq U_w$ and $Q \subseteq U_w$

… …

(iii) $|\,\boldsymbol{A}\,|^{\mathcal{M}_w} = F_w(\boldsymbol{A})$, if $\boldsymbol{A} \in N_E$

(iv-a) $|\,\boldsymbol{D}\ \boldsymbol{N}\,|^{\mathcal{M}_w} = |\,\boldsymbol{D}\,|^{\mathcal{M}_w}(|\,\boldsymbol{N}\,|^{\mathcal{M}_w})$
if $\boldsymbol{D}\ \boldsymbol{N}$ is a quantifier phrase, where $\boldsymbol{D}$ is a quantificational determiner and $\boldsymbol{N}$ is a count noun;

(iv-b) $|\,\boldsymbol{V}\ \boldsymbol{Q}\,|^{\mathcal{M}_w} = \lambda x.\ |\,\boldsymbol{Q}\,|^{\mathcal{M}_w}(\lambda y. |\,\boldsymbol{V}\,|^{\mathcal{M}_w}(y)(x))$
if $\boldsymbol{V}\ \boldsymbol{Q}$ is a predicate, where $\boldsymbol{V}$ is a transitive verb and $\boldsymbol{Q}$ is a quantifier phrase;

… …

Definition: extensional types Church (1940)

(i) The extensions of sentences are *of type t*;
(ii) those of terms are *of type e*;
(iii) if the extensions of an expression operates on extensions of some type a resulting in extensions of some type b, it is of type (a,b).

In other words, (i) t is the type of truth values; (ii) e is the type of individuals (or entities); (iii) (a,b) is the type of (total) functions from a to b. In this notation, the extensions of sentences, terms, coordinating conjunctions, predicates, transitives, quantificational phrases, and determiners are of types t; e; $(t\ (t,t))$; (e,t); $(e,(e,t))$; $((e,t),t)$; and $((e,t),((e,t),t))$, respectively.

Definition: replacements Fine (1977); Rabinowicz(1979)

Given (not necessarily distinct) possible worlds w and w', a *replacement* (of w by w') is a bijective function from the domain of individuals of w to the domain of individuals of w' (which must therefore have the same cardinality). Given a replacement ρ (from w to w'), the following recursive equations define corresponding functions ρ_a, for each type a:

- $\rho_e = \rho$;
- $\rho_t = \{(0,0),(1,1)\}$ [$= \lambda x.\ x$, where 'x' ranges over truth values];
- $\rho_{(a,b)} = \lambda f.\ \{(\rho_a(x), \rho_b(y) \mid f(x) = y\}$ [where 'f' ranges over functions of type (a,b)].

Digression: Logicality as Invariance Lindenbaum & Tarski (1935)

If f is an intension of type a (i.e. a function from W to extensions of type a), then f is (*replacement-*) *invariant* just in case for any replacements ρ and ρ' of w by some world w', it holds that $\rho_a(f(w)) = \rho'_a(f(w))$; *logical words* may then be characterised as lexical expressions $\boldsymbol{A}$ with invariant intensions: $\rho_{\tau(\boldsymbol{A})}(\| \boldsymbol{A} \|^{w}) = \rho'_{\tau(\boldsymbol{A})}(\| \boldsymbol{A} \|^{w})$, for any worlds w and replacements ρ and ρ' of w by some world w'.

(30) $|\textbf{every boy fancies Mary and Jane sulks}|^{\mathcal{M}_w}$

= $\vdash$ Jane sulks in $w \dashv \times \vdash \{x \mid x \text{ is a boy in } w\} \subseteq \{y \mid y \text{ fancies Mary in } w\} \dashv$

(31a) $|\boldsymbol{A}|^{\mathcal{M}_w} = \|\boldsymbol{A}\|^{w}$

(b) $\|\boldsymbol{A}\|^{w} = \|\boldsymbol{B}\|^{w}$ iff $|\boldsymbol{A}|^{\mathcal{M}_w} = |\boldsymbol{B}|^{\mathcal{M}_w}$

Definition
If w and w' are possible worlds and L is a language, then w is L-*indistinguishable* from w' – in symbols: $w \equiv_L w'$ – iff $\|\boldsymbol{A}\|^{w} = \|\boldsymbol{A}\|^{w'}$, for any expression $\boldsymbol{A}$ of L.

Definition
A language L is *discriminative* iff no two distinct possible worlds w and w' are L-indistinguishable.

(32) If $w \equiv_L w'$, then $\mathcal{M}_w = \mathcal{M}_{w'}$.

2.3 Intensionality

(33) **John thinks <u>Mary is home</u>.**
(34) **John thinks <u>Ann is pregnant</u>.**

(35) If: $\|\textbf{Mary is home}\|^{w} = \|\textbf{Ann is pregnant}\|^{w}$, Frege (1892)

then: $f(\|\textbf{Mary is home}\|^{w}) = f(\|\textbf{Ann is pregnant}\|^{w})$.

(36) $\|\textbf{thinks}\|^{w}(\|\boldsymbol{S}\|^{w})(\|\boldsymbol{T}\|^{w}) = \|\boldsymbol{T}\textbf{ thinks }\boldsymbol{S}\|^{w}$
(37) **John thinks General Beauregard Lee is a woodchuck.**
(38) **John thinks General Beauregard Lee is a whistlepig.**

Definition: intensional types Montague (1970)

(i) The extensions of sentences are of *type t*;
(ii) those of terms are of type *e*;
(iii) if the extension of an expression operates on extensions of some type a resulting in extensions of some type b, it is of type (a,b).
(iv) it is of type $((s,a),b)$ if it operates on intensions of expressions whose extensions are of type a, resulting in extensions of type b.

In other words, (iv) (s,a) is the type of (total) functions from Logical Space to a. In this notation, the extension of an attitude verb is of type $((s,t),(e,t))$.

(22f) If $\boldsymbol{V}\,\boldsymbol{S}$ is a predicate, where $\boldsymbol{V}$ is an attitude (= clause-embedding) verb and $\boldsymbol{S}$ is a sentence, then:

$$\|\boldsymbol{V}\,\boldsymbol{S}\|^{w} = \|\boldsymbol{V}\|^{w}(\|\boldsymbol{S}\|^{w}).$$

(39) $\| \textbf{Jane doubts that every boy fancies Mary} \|^{w}$

= $\| \textbf{doubts} \|^{w}(\| \textbf{every boy fancies Mary} \|)(\| \textbf{Jane} \|^{w})$ … by (22c&f)

(40) $[\lambda p.\ \lambda x.\ \| \textbf{doubt} \|^{w}(p)(x) \cap \| \textbf{believe} \|^{w}(p)(x)] = \emptyset$

(iv-c) $|\, \boldsymbol{V\, S} \,|^{\mathcal{M}_w} = |\, \boldsymbol{V} \,|^{\mathcal{M}_w}(\lambda w'.\, |\, \boldsymbol{S} \,|^{\mathcal{M}_{w'}})$
if $\boldsymbol{V\, S}$ is a predicate, where $\boldsymbol{V}$ is an attitude verb and $\boldsymbol{S}$ is a clausal complement.

Definition
Given a possible world w^* and a language L, the *intensional material model* (*for L based on w**) is the quadruple $\hat{\mathcal{M}}_{w^*} = (W, \hat{U}, w^*, \hat{F})$ consisting of the set W of all possible worlds; the *domain* function $\hat{U}$ assigning to each possible world w the domain of individuals U_w; the world w^* itself; and the lexical interpretation function $\hat{F}$ which assigns to every non-logical lexical expression $\boldsymbol{A}$ of L the intension of $\boldsymbol{A}$ at w.

3. Model-theoretic Semantics

3.1 Extensional Model Space

Definition

Given a language L, a *formal model* (*for L*) is a pair $\mathcal{M} = (U, F)$ consisting of a non-empty set U (= the *universe of* $\mathcal{M}$) and a function F which assigns to every non-logical lexical expression $\boldsymbol{A}$ of L a U-extension of type $\tau_L(\boldsymbol{A})$.

Observations
Let U_w be the domain of individuals of some world w, $X \in U_w$ an invariant extension of some type a, and ρ a bijection from U_w to a set U of the same cardinality. Then:

(*) $\rho'_a(X) = \rho''_a(\rho_a(X))$, for any bijections ρ' and ρ'' from U_w and U to some set U' of the same cardinality, respectively;

(**) $\rho_a(X) = \rho'_a(X)$, for any bijection ρ' from U_w to U.

(41) If ρ is a model-isomorphism from the material model $\mathcal{M}_w = (U_w, F_w)$ (of a language L) to the formal model $\mathcal{M} = (U, F)$, then the *extension* $\llbracket \boldsymbol{A} \rrbracket^{\mathcal{M}}$ *of* an expression $\boldsymbol{A}$ is determined by the following induction on $\boldsymbol{A}$'s complexity:

(i) $\llbracket \boldsymbol{A} \rrbracket^{\mathcal{M}} = |\, \boldsymbol{A} \,|^{\mathcal{M}_w}$ $[= \| \boldsymbol{A} \|^{w}]$, if $\boldsymbol{A}$ is a truth-functional lexical item;

(ii) $\llbracket \boldsymbol{A} \rrbracket^{\mathcal{M}} = \rho_{\tau(\boldsymbol{A})}(|\, \boldsymbol{A} \,|^{\mathcal{M}_w})$, if $\boldsymbol{A}$ is a combinatorial lexical item;

(iii) $\llbracket \boldsymbol{A} \rrbracket^{\mathcal{M}} = F(\boldsymbol{A})$ $[= \rho_{\tau(\boldsymbol{A})}(F_w(\boldsymbol{A}))]$, if $\boldsymbol{A} \in N_L$;

(iv) $\llbracket \boldsymbol{A} \rrbracket^{\mathcal{M}} = \rho_{\tau(\boldsymbol{A})}(G(\rho^{-1}_{\tau(\boldsymbol{B}_1)}(\llbracket \boldsymbol{B}_1 \rrbracket^{\mathcal{M}}), \ldots, \rho^{-1}_{\tau(\boldsymbol{B}_n)}(\llbracket \boldsymbol{B}_n \rrbracket^{\mathcal{M}})))$, if $\boldsymbol{A}$ is a complex expression with immediate constituents $\boldsymbol{B}_1, \ldots, \boldsymbol{B}_n$ such that $|\, \boldsymbol{A} \,|^{\mathcal{M}_w} = G(|\, \boldsymbol{B}_1 \,|^{\mathcal{M}_w}, \ldots, |\, \boldsymbol{B}_n \,|^{\mathcal{M}_w})$

(42) Let ρ be a model isomorphism from $\mathcal{M}$ to $\tilde{\mathcal{M}}$. Then

$$\rho_{\tau(\boldsymbol{A})}([\![\boldsymbol{A}]\!]^{\mathcal{M}}) = [\![\boldsymbol{A}]\!]^{\tilde{\mathcal{M}}},$$

for all expressions ***A*** of *L*.

(43) If $\mathcal{M}_w$ is a material model for a language *L* and ***A*** is an expression of *L*, then

$$[\![\boldsymbol{A}]\!]^{\mathcal{M}_w} = |\,\boldsymbol{A}\,|^{\mathcal{M}_w} = \|\,\boldsymbol{A}\,\|^{w}.$$

cf. (31a)

(44) If the formal model $\mathcal{M}$ (for a language *L*) represents the material model $\mathcal{M}_w$, then

$$[\![\boldsymbol{S}]\!]^{\mathcal{M}} = [\![\boldsymbol{S}]\!]^{\mathcal{M}_w},$$

for all sentences ***S*** of *L*.

Definition
If *L* is a language *L*, then (*L*'s) *Ersatz Space* is the class of all formal models $\mathcal{M} = (U,F)$ of *L* such that *U* is a pure set and $\mathcal{M}$ represents a material model.

(45) $|\,\mathcal{M}_0\,|_{\cong} := \{\mathcal{M} \mid \mathcal{M}$ is an ersatz model for L & $\mathcal{M} \cong \mathcal{M}_0\}$,

(46) If $w \neq w'$, then $\mathcal{M}_w \neq \mathcal{M}_{w'}$.

(47) If $\mathcal{M}_w \neq \mathcal{M}_{w'}$, then $|\,\mathcal{M}_w\,|_{\cong} \neq |\,\mathcal{M}_{w'}\,|_{\cong}$.

(48a) $\|\,\mathbf{w}\,\|^{w} = w$; (b) $\|\,\mathbf{W}\,\|^{w} = \lambda w.\ w' = w$.

Definitions

- A language *L* is *complete* iff no two distinct possible worlds w and w' are *L*-equivalent.
- If w and w' are possible worlds and *L* is a language, then w is *L-equivalent* to w' – in symbols: $w \approx_L w'$ – iff $\|\,\boldsymbol{S}\,\|^{w} = \|\,\boldsymbol{S}\,\|^{w'}$, for any declarative sentence ***S*** of *L*.

(49a) If $w \equiv_L w'$, then $w \approx_L w'$.
(b) If *L* is complete, then *L* is discriminative.
(c) If *L* is complete and $w \neq w'$, then $|\,\mathcal{M}_w\,|_{\cong} \neq |\,\mathcal{M}_{w'}\,|_{\cong}$.

(50) If ***A*** and ***B*** are expressions of a language *L*, then $\|\,\boldsymbol{A}\,\| = \|\,\boldsymbol{B}\,\|$ iff $[\![\boldsymbol{A}]\!]^{\mathcal{M}} = [\![\boldsymbol{B}]\!]^{\mathcal{M}}$, for all members $\mathcal{M}$ of *L*'s Ersatz Space.

(51) ***N*** is a hyponym of ***N'***

iff $\|\,\boldsymbol{N}\,\|^{w} \subseteq \|\,\boldsymbol{N'}\,\|^{w}$, for all worlds w — by definition of *hyponym* (cf. Section 2.1)

iff $[\![\boldsymbol{N}]\!]^{\mathcal{M}_w} \subseteq [\![\boldsymbol{N'}]\!]^{\mathcal{M}_w}$, for all material models $\mathcal{M}_w$ — by (43)

iff $\rho([\![\boldsymbol{N}]\!]^{\mathcal{M}_w}) \subseteq \rho([\![\boldsymbol{N'}]\!]^{\mathcal{M}_w})$, for all material models $\mathcal{M}_w$ and all bijective functions ρ from U_w to some pure set U of the same cardinality — ρ preserves Boolean structure

iff $[\![\boldsymbol{N}]\!]^{\mathcal{M}} \subseteq [\![\boldsymbol{N'}]\!]^{\mathcal{M}}$, for all material models $\mathcal{M}_w$ and all $\mathcal{M} \in |\,\mathcal{M}_w\,|_{\cong}$ — by (44)

iff $[\![\boldsymbol{N}]\!]^{\mathcal{M}} \subseteq [\![\boldsymbol{N'}]\!]^{\mathcal{M}}$, for all ersatz models $\mathcal{M}$ — by definition of *ersatz model*

(52) For any expression $\boldsymbol{A}$ of E and any formal model $\mathcal{M} = (U,F)$ for E, the *extension of* $\boldsymbol{A}$ *relative to* $\mathcal{M}$ – $[\![\boldsymbol{A}]\!]^{\mathcal{M}}$ – is determined by the following induction (on the grammatical complexity of $\boldsymbol{A}$):

(i-a) $[\![\mathbf{and}]\!]^{\mathcal{M}} = \lambda u.\ \lambda v.\ u \times v$ … where $u \in \{0,1\}$ and $v \in \{0,1\}$

… …

(ii-a) $[\![\mathbf{every}]\!]^{\mathcal{M}} = \lambda P.\ \lambda Q.\ \vdash P \subseteq Q \dashv$ where $P \subseteq U$ and $Q \subseteq U$

… …

(iii) $[\![\boldsymbol{A}]\!]^{\mathcal{M}} = F_w(\boldsymbol{A})$, if $\boldsymbol{A} \in N_E$

(iv-a) $[\![\boldsymbol{DN}]\!]^{\mathcal{M}} = [\![\boldsymbol{D}]\!]^{\mathcal{M}}([\![\boldsymbol{N}]\!]^{\mathcal{M}})$
if $\boldsymbol{D}\ \boldsymbol{N}$ is a quantifier phrase, where $\boldsymbol{D}$ is a quantificational determiner and $\boldsymbol{N}$ is a count noun;

(iv-b) $[\![\boldsymbol{V}\ \boldsymbol{Q}]\!]^{\mathcal{M}} = \lambda x.\ [\![\boldsymbol{Q}]\!]^{\mathcal{M}}(\lambda y.[\![\boldsymbol{V}]\!]^{\mathcal{M}}(y)(x)))$
if $\boldsymbol{V}\ \boldsymbol{Q}$ is a predicate, where $\boldsymbol{V}$ is a transitive verb, $\boldsymbol{Q}$ is a quantifier phrase, and x and y are elements of U.

… …

Digression: Meaning Postulates (and other Constraints) Carnap (1952)

(53) **No bachelor is married.**
(54) **Every bachelor is not married.**
(55) **Some bachelor is married.**
(56) $F(\mathbf{bachelor}) \cap F(\mathbf{married}) = \emptyset$
(57) $F^*(\mathbf{sulks}) = \lambda Q.\ Q(F(\mathbf{sulks}))$ cf. Zimmermann (1985)
(58) $F^*(\mathbf{sulks}) = \lambda Q.\ Q(F(X))$, for some U-extension of type (et)
(59) U is countable.

Definition
Let $\mathscr{K}$ be a class of pure models for a language L and $\boldsymbol{S}$ and $\boldsymbol{S}'$ be sentences of L.

- $\boldsymbol{S}$ is $\mathscr{K}$*-valid* – in symbols: $\vDash_{\mathscr{M}} \boldsymbol{S}$ – iff
  $[\![\boldsymbol{S}]\!]^{\mathcal{M}} = 1$, for all $\mathcal{M} \in \mathscr{K}$.
- $\boldsymbol{S}$ $\mathscr{K}$*-entails* $\boldsymbol{S}'$ – in symbols: $\boldsymbol{S} \vDash_{\mathscr{M}} \boldsymbol{S}'$ – iff
  $\{\mathcal{M} \in \mathscr{K} \mid [\![\boldsymbol{S}]\!]^{\mathcal{M}} = 1\} \subseteq \{\mathcal{M} \in \mathscr{K} \mid [\![\boldsymbol{S}']\!]^{\mathcal{M}} = 1\}$

(60) If $\mathcal{M} \cong \mathcal{M}'$, then $\mathcal{M} \in \mathscr{K}$ iff $\mathcal{M}' \in \mathscr{K}$.

Digression: Permutation Argument Putnam (1977)

(61) For any expression $\boldsymbol{A}$ of E and any formal model $\mathcal{M} = (U,F)$ for E, the *permuted extension of* $\boldsymbol{A}$ *relative to* $\mathcal{M}$ – $/\!/\boldsymbol{A}/\!/^{\mathcal{M}}$ – is determined by the following induction (on the grammatical complexity of $\boldsymbol{A}$):

(i-a) $/\!/\mathbf{and}/\!/^{\mathcal{M}} = \lambda u.\ \lambda v.\ u \times v$ … where $u \in \{0,1\}$ and $v \in \{0,1\}$

… …

(ii-a) $/\!/\mathbf{every}/\!/^{\mathcal{M}} = \lambda P.\ \lambda Q.\ \vdash P \subseteq Q \dashv$ where $P \subseteq U$ and $Q \subseteq U$

… …

(iii) $/\!/\boldsymbol{A}/\!/^{\mathcal{M}} = \pi_{\tau(\boldsymbol{A})}(F_w(\boldsymbol{A}))$, if $\boldsymbol{A} \in N_E$

(iv-a) $/\!/\boldsymbol{D}\ \boldsymbol{N}/\!/^{\mathcal{M}} = /\!/\boldsymbol{D}/\!/^{\mathcal{M}}(/\!/\boldsymbol{A}/\!/^{\mathcal{M}})$
if $\boldsymbol{D}$ is a quantificational determiner and $\boldsymbol{N}$ is a count noun;

… …

3.2 Intensional Model Space

Definition

A *formal ontology* is a pair $(W,\hat{U})$, where W is a non-empty set (the *worlds* according to $(W,\hat{U})$) and $\hat{U}$ is a function with domain W such that $\hat{U}(w)$ is a non-empty set whenever $w \in W$ [= the individuals of w, according to $(W,\hat{U})$].

Definition

Given a language L, an *intensional formal model* (*for* L) is a quadruple $\hat{\mathcal{M}} = (W,\hat{U},w^*,\hat{F})$, where $(W,\hat{U})$ is a formal ontology; a member w^* of W (= the *actual* world according to $\hat{\mathcal{M}}$); and a function $\hat{F}$ which assigns to every non-logical lexical expression $\boldsymbol{A}$ of L a $(W,\hat{U})$-intension of type $\tau_L(\boldsymbol{A})$ (= the lexical interpretation function according to $\hat{\mathcal{M}}$).

$(5\hat{2})$ For any expression $\boldsymbol{A}$ of $\hat{E}$, any intensional formal model $\hat{\mathcal{M}} = (W,\hat{U},w^*,\hat{F})$ for $\hat{E}$, and any world w (according to $\hat{\mathcal{M}}$), the *extension* $\llbracket \boldsymbol{A} \rrbracket^{\hat{\mathcal{M}},w}$ *of* $\boldsymbol{A}$ *at* w *relative to* $\hat{\mathcal{M}}$ is determined by the following induction:

(i-a) $\llbracket \mathbf{and} \rrbracket^{\hat{\mathcal{M}},w} = \lambda u.\ \lambda v.\ u \times v$ … where $u \in \{0,1\}$ and $v \in \{0,1\}$

… …

(ii-a) $\llbracket \mathbf{every} \rrbracket^{\hat{\mathcal{M}},w} = \lambda P.\ \lambda Q.\ \vdash P \subseteq Q \dashv$ … where $P \subseteq \hat{U}(w)$ and $Q \subseteq \hat{U}(w)$

… …

(ii-c) $\llbracket \mathbf{necessarily} \rrbracket^{\hat{\mathcal{M}},w} = \lambda p.\ \vdash p = W \dashv$ … where $p \subseteq W$

… …

(iii) $\llbracket \boldsymbol{A} \rrbracket^{\hat{\mathcal{M}},w} = \hat{F}(\mathbf{A})(w)$, if $\boldsymbol{A} \in N_E$

(iv-a) $\llbracket \boldsymbol{DN} \rrbracket^{\hat{\mathcal{M}},w} = \llbracket \boldsymbol{D} \rrbracket^{\hat{\mathcal{M}},w}(\llbracket \boldsymbol{N} \rrbracket^{\hat{\mathcal{M}},w})$
if $\boldsymbol{D}$ $\boldsymbol{N}$ is a quantifier phrase, where $\boldsymbol{D}$ is a quantificational determiner and $\boldsymbol{N}$ is a count noun;

… …

(iv-c) if $\boldsymbol{V}$ $\boldsymbol{S}$ is a predicate, where $\boldsymbol{V}$ is an attitude verb and $\boldsymbol{S}$ is a clausal complement,

$\llbracket \boldsymbol{V\ S} \rrbracket^{\hat{\mathcal{M}},w} = \llbracket \boldsymbol{S} \rrbracket^{\hat{\mathcal{M}},w}(\lambda w'.\ \llbracket \boldsymbol{S} \rrbracket^{\hat{\mathcal{M}},w'})$;

(iv-d) if $\boldsymbol{A}$ $\boldsymbol{S}$ is a sentence, where $\boldsymbol{A}$ is a sentential adverb and $\boldsymbol{S}$ is a sentence,

$\llbracket \boldsymbol{A\ S} \rrbracket^{\hat{\mathcal{M}},w} = \llbracket \boldsymbol{A} \rrbracket^{\hat{\mathcal{M}},w}(\lambda w'.\ \llbracket \boldsymbol{S} \rrbracket^{\hat{\mathcal{M}},w'})$;

… …

(62a) $\| \textbf{Jane doubts that every boy fancies Mary} \|^{w_0}$

iff $(\| \textbf{Jane} \|^{w_0}, \{w' \in L \mid \| \textbf{boy} \|^{w'} \subseteq \{x \in U_{w'} \mid (x, \| \textbf{Mary} \|^{w'}) \in \| \textbf{fancies} \|^{w'}\}) \in \| \textbf{doubts} \|^{w_0}$ cf. (39)

(b) $\llbracket \textbf{Jane doubts that every boy fancies Mary} \rrbracket^{\hat{\mathcal{M}},w} = 1$

iff $(\llbracket \mathbf{Jane} \rrbracket^{\hat{\mathcal{M}},w}, \{w' \in L \mid \llbracket \mathbf{boy} \rrbracket^{\hat{\mathcal{M}},w'} \subseteq \{x \in \hat{U}(w') \mid (x, \llbracket \mathbf{Mary} \rrbracket^{\hat{\mathcal{M}},w'}) \in \llbracket \mathbf{fancies} \rrbracket^{\hat{\mathcal{M}},w'}\}) \in \llbracket \mathbf{doubts} \rrbracket^{\hat{\mathcal{M}},w}$

(c) $\llbracket \mathbf{Jane\ doubts\ that\ every\ boy\ fancies\ Mary} \rrbracket^{\hat{\mathcal{M}}} = 1$

iff $(\llbracket \mathbf{Jane} \rrbracket^{\hat{\mathcal{M}}}, \{w' \in L \mid \llbracket \mathbf{boy} \rrbracket^{\hat{\mathcal{M}},w'} \subseteq \{x \in \hat{U}(w') \mid (x, \llbracket \mathbf{Mary} \rrbracket^{\hat{\mathcal{M}},w'}) \in \llbracket \mathbf{fancies} \rrbracket^{\hat{\mathcal{M}},w'}\}) \in \llbracket \mathbf{doubts} \rrbracket^{\hat{\mathcal{M}},w}$

(63) $\llbracket \mathbf{Necessarily}\ \boldsymbol{S} \rrbracket^{\hat{\mathcal{M}},w} = \vdash \lambda w'.\ \llbracket \boldsymbol{S} \rrbracket^{\hat{\mathcal{M}},w'} = W \dashv$

Definition

If $(W,\hat{U})$ is an formal ontology, then a $(W,\hat{U})$*-replacement* is a pair $\rho = (\rho^s, \rho^e)$ of functions with domain W such that:

- ρ^s is a bijection on W;
- $\rho^e(w)$ is a bijection on $\hat{U}(\rho^s(w))$, whenever $w \in W$.

- $\rho_e = \rho^e(w)$;
- $\rho_t = \{(0,0),(1,1)\}$;
- $\rho_{(a,b)} = \lambda f.\ \{(\rho_a(x), \rho_b(y)) \mid f(x) = y\}$;
  [where 'f' ranges over $(W,\hat{U},w)$-extensions of type (a,b)];
- $\rho_{(s,a)} = \lambda f.\ \{(\rho^s(w'), \rho_a(y)) \mid f(w') = y\}$
  [where 'f' ranges over $(W,\hat{U},w)$-extensions of type (s,a)].

Definition

If $\hat{\mathcal{M}}_1 = (W_1, \hat{U}_1, w_1, \hat{F}_1)$ and $\hat{\mathcal{M}}_2 = (W_2, \hat{U}_2, w_2, \hat{F}_2)$ are intensional formal models for a language L, then a *model isomorphism from* $\hat{\mathcal{M}}_1$ *to* $\hat{\mathcal{M}}_2$ is a pair $\rho = (\rho^s, \rho^e)$ of bijections from W_1 to W_2 and from $\bigcup \hat{U}_1$ to $\bigcup \hat{U}_2$, respectively, such that

- $\rho^s(w_1) = w_2$;
- $\rho_{\tau(\boldsymbol{A})}(\hat{F}_1(\boldsymbol{A})(w)) = \hat{F}_2(\boldsymbol{A})(\rho^s(w))$, whenever $w \in W_1$ and $\boldsymbol{A} \in N_L$.

$(\hat{42})$ Let ρ be a model isomorphism from $\hat{\mathcal{M}}_1$ to $\hat{\mathcal{M}}_2$. Then

$$\rho_{\tau(\boldsymbol{A})}(\llbracket \boldsymbol{A} \rrbracket^{\mathcal{M}}) = \llbracket \boldsymbol{A} \rrbracket^{\tilde{\mathcal{M}}},$$

for all expressions $\boldsymbol{A}$ of L.

(64)

$$\text{If}\left\{\begin{array}{l}\text{(a) } \llbracket \boldsymbol{A} \rrbracket^{\mathcal{M}} = \llbracket \boldsymbol{B} \rrbracket^{\mathcal{M}} \\ \text{(b) } \llbracket \boldsymbol{A} \rrbracket^{\mathcal{M}} \subseteq \llbracket \boldsymbol{B} \rrbracket^{\mathcal{M}} \\ \text{(c) } \llbracket \boldsymbol{A} \rrbracket^{\mathcal{M}} = 1\end{array}\right\}\text{, for all pure models } \mathcal{M} \text{ for } L,$$

$$\text{then}\left\{\begin{array}{l}\llbracket \boldsymbol{A} \rrbracket^{\hat{\mathcal{M}}} = \llbracket \boldsymbol{B} \rrbracket^{\hat{\mathcal{M}}} \\ \llbracket \boldsymbol{A} \rrbracket^{\hat{\mathcal{M}}} \subseteq \llbracket \boldsymbol{B} \rrbracket^{\hat{\mathcal{M}}} \\ \llbracket \boldsymbol{A} \rrbracket^{\hat{\mathcal{M}}} = 1\end{array}\right\}\text{, for all pure intensional models } \hat{\mathcal{M}} \text{ for } L.$$

($64_{\mathscr{K}}$)

$$\text{If}\left\{\begin{array}{l}\text{(a) } \llbracket \boldsymbol{A} \rrbracket^{\mathcal{M}} = \llbracket \boldsymbol{B} \rrbracket^{\mathcal{M}} \\ \text{(b) } \llbracket \boldsymbol{A} \rrbracket^{\mathcal{M}} \subseteq \llbracket \boldsymbol{B} \rrbracket^{\mathcal{M}} \\ \text{(c) } \llbracket \boldsymbol{A} \rrbracket^{\mathcal{M}} = 1\end{array}\right\}\text{, for all } \mathcal{M} \in \mathscr{K},$$

$$\text{then}\left\{\begin{array}{l}\text{(a) } \llbracket \boldsymbol{A} \rrbracket^{\hat{\mathcal{M}}} = \llbracket \boldsymbol{B} \rrbracket^{\hat{\mathcal{M}}} \\ \text{(b) } \llbracket \boldsymbol{A} \rrbracket^{\hat{\mathcal{M}}} \subseteq \llbracket \boldsymbol{B} \rrbracket^{\hat{\mathcal{M}}} \\ \text{(c) } \llbracket \boldsymbol{A} \rrbracket^{\hat{\mathcal{M}}} = 1\end{array}\right\}\text{, for all } \hat{\mathcal{M}} \in \hat{\mathscr{K}}.$$

Definition

Let **S** be a sentences of a language *L*, and let $\hat{\mathcal{M}}$ and $\hat{\mathscr{K}}$ be a pure model and a class of pure models for *L*, respectively.

- **S** is *valid in* $\hat{\mathcal{M}}$ – in symbols: $\hat{\mathcal{M}} \vDash \boldsymbol{S}$ – iff $^\wedge\llbracket \boldsymbol{S} \rrbracket^{\hat{\mathcal{M}}} = W$.
- **S** is $\hat{\mathscr{K}}$*-valid* – in symbols: $\vDash_{\hat{\mathscr{K}}}$ **S** – iff **S** is valid in every member $\hat{\mathscr{K}}$.

(65) Let $\hat{\mathscr{K}}$ be the class of pure intensional models (for *L*) determined by some postulate system.

(a) If $\hat{\mathcal{M}}_1 \in \hat{\mathscr{K}}$ and ρ is a model isomorphism from $\hat{\mathcal{M}}_1$ to a pure intensional model $\hat{\mathcal{M}}_2$, then $\hat{\mathcal{M}}_2 \in \hat{\mathscr{K}}$.

(b) If $\hat{\mathcal{M}}_2$ is a variant of a pure intensional model $\hat{\mathcal{M}}_1$, then $\hat{\mathcal{M}}_1 \in \hat{\mathscr{K}}$ iff $\hat{\mathcal{M}}_2 \in \hat{\mathscr{K}}$.

(c) If a sentence **S** of *L* is true in all members of $\hat{\mathscr{K}}$, then **S** is $\hat{\mathscr{K}}$-valid (and hence valid in all members of $\mathscr{K}$).

Digression: Meaning Postulates in Intensional Model Space Zimmermann (1999)

(66) **Nothing both rises and falls**.

(67) **No one both doubts and believes anything .**

(68) $[\lambda p.\, \lambda x.\, \llbracket \mathbf{doubt} \rrbracket^{\hat{\mathcal{M}}}(p)(x) \cap \llbracket \mathbf{doubt} \rrbracket^{\hat{\mathcal{M}}}(p)(x)] = \emptyset$ cf. (40)

(69a) **Jones is a professional soccer player.**

(b) **Jones is not an amateur soccer player.**

(70) $[\lambda P.\, \lambda x.\, [\![\textbf{professional}]\!]^{\hat{\mathcal{M}}}(P)(x) \cap [\![\textbf{amateur}]\!]^{\hat{\mathcal{M}}}(P)(x)] = \emptyset$,

(71) **No one is both a professional *N* and an amateur *N*.**

(71) For any $w \in W$, $\hat{U}(w)$ is countable.

(72) W is infinite.

(73) If $\hat{\mathcal{M}} \cong \hat{\mathcal{M}}'$, then $\hat{\mathcal{M}} \in \mathscr{K}$ iff $\hat{\mathcal{M}}' \in \hat{\mathscr{K}}$.

(74) If $\hat{\mathcal{M}}$ is a variant of $\hat{\mathcal{M}}'$, then $\hat{\mathcal{M}} \in \hat{\mathscr{K}}$ iff $\hat{\mathcal{M}}' \in \hat{\mathscr{K}}$.

(75a) **teacher** : **smoker**

(b) **Mary is asleep** : **Jane sulks**

(c) **expensive** : **green**

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