Fregean Compositionality

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1. *Background*

(1*a*) John seeks a unicorn. suface

(*b*) $\mathfrak{P}(\Delta_{John},\Delta_{seeks\ a\ unicorn})$ unspecific

(*c*) $\mathfrak{Q}(\Delta_{a\ unicorn},\Delta_{John\ seeks\ \_\_})$ specific

(2) $[\![\mathfrak{F}(\Delta_1,\ldots,\Delta_n)]\!]^{\wedge} = [\![\mathfrak{F}]\!]([\![\Delta_1]\!]^{\wedge},\ldots,[\![\Delta_1]\!]^{\wedge})$

(3) $[\![\mathfrak{P}]\!](x,P)(i) = P_i(x_i)$ $= P(i)(x(i))$

(4) $[\![\mathfrak{P}(\Delta_{John},\Delta_{seeks\ a\ unicorn})]\!]^{i} = 1$

iff $[\![\mathfrak{P}(\Delta_{John},\Delta_{seeks\ a\ unicorn})]\!]^{\wedge}(i) = 1$ notation: $[\![\Delta]\!]^{i} = [\![\Delta]\!]^{\wedge}(i)$

iff $[\![\mathfrak{P}]\!]([\![\Delta_{John}]\!]^{\wedge},[\![\Delta_{seeks\ a\ unicorn}]\!]^{\wedge})(i) = 1$ (2)

iff $[\![\Delta_{seeks\ a\ unicorn}]\!]^{\wedge}(i)([\![\Delta_{John}]\!]^{\wedge}(i)) = 1$ (3)

iff $[\![\Delta_{seeks\ a\ unicorn}]\!]^{i}([\![\Delta_{John}]\!]^{i}) = 1$ notation

(5) $[\![\mathfrak{O}]\!](R,Q)(i) = R_i(Q)$,

(6) $[\![\mathfrak{P}(\Delta_{John},\Delta_{seeks\ a\ unicorn})]\!]^{i} = 1$

iff $[\![\Delta_{seeks\ a\ unicorn}]\!]^{\wedge}(i)([\![\Delta_{John}]\!]^{\wedge}(i)) = 1$ (4)

iff $[\![\mathfrak{O}(\Delta_{seeks},\Delta_{a\ unicorn})]\!]^{\wedge}(i)([\![\Delta_{John}]\!]^{\wedge}(i)) = 1$ def. $\Delta_{seeks\ a\ unicorn}$

iff $[\![\mathfrak{O}]\!]([\![\Delta_{seeks}]\!]^{\wedge},[\![\Delta_{a\ unicorn}]\!]^{\wedge})(i)([\![\Delta_{John}]\!]^{\wedge}(i)) = 1$ (2)

iff $[\![\Delta_{seeks}]\!]^{\wedge}(i)([\![\Delta_{a\ unicorn}]\!]^{\wedge})([\![\Delta_{John}]\!]^{\wedge}(i)) = 1$ (5)

iff $[\![\Delta_{seeks}]\!]^{i}([\![\Delta_{John}]\!]^{i},[\![\Delta_{a\ unicorn}]\!]^{\wedge}) = 1$ notation

2. *Extensional compositionality*

(7) $[\![\mathfrak{P}(\Delta_1,\Delta_2)]\!]^{i} = [\![\Delta_2]\!]^{i}([\![\Delta_1]\!]^{i})$ by (2) & (3)

(8) $[\![\mathfrak{O}(\Delta_1,\Delta_2)]\!]^{i} = [\![\Delta_1]\!]^{i}([\![\Delta_2]\!]^{\wedge})$ by (5) & (3)

(9) *Definition*

A construction $\mathfrak{F}$ (of n places) is *extensional* iff, at any point $i \in D_s$, the extension of an expression of the form $\mathfrak{F}(\Delta_1,\ldots,\Delta_n)$ at i, is determined by the extensions of its immediate parts at i, i.e.:

- $[\![\mathfrak{F}(\Delta_1,\ldots,\Delta_n)]\!]^{i} = [\![\mathfrak{F}(\Delta'_1,\ldots,\Delta'_n)]\!]^{i}$ whenever $[\![\Delta_1]\!]^{i} = [\![\Delta'_1]\!]^{i},\ldots,[\![\Delta_n]\!]^{i} = [\![\Delta'_n]\!]^{i}$

for any (appropriate) expressions $\Delta_1, \Delta'_1,\ldots, \Delta_n, \Delta'_n$.

(10) *Definition*

A semantic operation $[\![\mathfrak{F}]\!]$ is *extensional* iff for any $i \in D_s$:

- $[\![\mathfrak{F}]\!](x_1,\ldots,x_n) = [\![\mathfrak{F}]\!](x'_1,\ldots,x'_n)$ whenever $x_1(i) = x'_1(i),\ldots,x_n(i) = x'_n(i)$

for any (appropriate) intensions $x_1, x'_1,\ldots, x_n, x'_n$.

(11) *Facts*

(a) $\mathbb{F}$ is extensional iff for any $i \in D_s$ there is an operation $[\![\mathbb{F}]\!]^i$ such that

- $[\![\mathbb{F}(\Delta_1,\ldots,\Delta_n)]\!]^i = [\![\mathbb{F}]\!]^i([\![\Delta_1]\!]^i,\ldots,[\![\Delta_n]\!]^i)$,

for any (appropriate) expressions $\Delta_1,\ldots,\Delta_n$.

(b) $[\![\mathbb{F}]\!]$ is extensional iff for any $i \in D_s$ there is an operation $[\![\mathbb{F}]\!]^i$ such that

- $[\![\mathbb{F}]\!](x_1,\ldots,x_n)(i) = [\![\mathbb{F}]\!]^i(x_1(i),\ldots,x_n(i))$

for any (appropriate) intensions $x_1,\ldots,x_n$.

(12) *Definition*

A semantic operation $[\![\mathbb{F}]\!]$ is *uniformly extensional* iff there is an operation $[\![\mathbb{F}]\!]_*$ such that, for any $i \in D_s$:

- $[\![\mathbb{F}]\!](x_1,\ldots,x_n)(i) = [\![\mathbb{F}]\!]_*(x_1(i),\ldots,x_n(i))$

for any (appropriate) intensions $x_1,\ldots,x_n$.

(13) $$[\![\mathbb{F}^{\#}]\!](p,q)(i) = \begin{cases} min(p(i),q(i)), \text{ if } i \neq i^{\#} \\ max(p(i^{\#}),q(i^{\#})), \text{ otherwise} \end{cases}$$

(14) $$[\![\mathbb{F}^{\#\#}]\!](\varphi,\psi)(i)(j) = \begin{cases} max(\varphi(j),\psi(j)), \text{ if } \varphi(i) = \psi(i) = 1 \\ min(\varphi(j),\psi(j)), \text{ otherwise} \end{cases}$$

3. *Fregean Intensionality*

(15*a*) John thinks that Mary is sick. $\nLeftrightarrow$
(*b*) John thinks that 2+2=5. Frege's (1892) substitution argument

(16*a*) John seeks a French restaurant that serves bouillabaisse. $\overset{?}{\Leftrightarrow}$
(*b*) John seeks a French restaurant.

(17*a*) John seeks a book on Mary's life. $\nLeftrightarrow$
(*b*) John seeks a movie on Mary's life.

(18*a*) $[\![\mathbb{F}(\Delta_1,\Delta_2)]\!]^\wedge = [\![\mathbb{F}]\!]([\![\Delta_1]\!]^\wedge,[\![\Delta_2]\!]^\wedge)$ [intensional] compositionality

(*b*) $[\![\mathbb{F}(\Delta_1,\Delta_2)]\!]^i = [\![\mathbb{F}]\!]_*([\![\Delta_1]\!]^i,[\![\Delta_2]\!]^i)$ [uniform] extensionality

(*c*) $[\![\mathbb{F}(\Delta_1,\Delta_2)]\!]^i = [\![\mathbb{F}]\!]_+([\![\Delta_1]\!]^i,[\![\Delta_1]\!]^\wedge)$ Fregean intensionality

(19) *Definition*

An n-place semantic operation $[\![\mathbb{F}]\!]$ is *(uniformly) extensional in K* ($\subseteq\{1,\ldots,n\}$) iff there is an operation $[\![\mathbb{F}]\!]_+$ such that, for any $i \in D_s$ and $x_1,\ldots,x_n$

- $[\![\mathbb{F}]\!](x_1,\ldots,x_n)(i) = [\![\mathbb{F}]\!]_+(X_1^{[i]},\ldots,X_n^{[i]})$

where (for $k \leq n$): $X_k^{[i]} = \begin{cases} x_k(i) \text{ if } i \in K \\ x_k \text{ otherwise} \end{cases}$

(20) $$[\![\mathbb{G}^{\#}]\!](p)(i) = \begin{cases} 1 \text{ if } p(i) = p(i^{\#}) \\ 0, \text{ otherwise} \end{cases}$$ Zimmermann & Sternefeld (2013: 197, fn. 27)

4. *The Hierarchy of Intensions*

(21*a*) Syd sees that Emily plays.

(*b*) $\mathfrak{P}(\Delta_{Syd}, \mathcal{A}(\Delta_{sees}, \Delta_{Emily\ plays}))$

(*c*) $[\![\mathfrak{P}(\Delta_{Syd}, \mathcal{A}(\Delta_{sees}, \Delta_{Emily\ plays}))]\!]^{\wedge}$

$= [\![\mathfrak{P}]\!]([\![\Delta_{Syd}]\!]^{\wedge}, [\![\mathcal{A}]\!]([\![\Delta_{sees}]\!]^{\wedge}, [\![\Delta_{Emily\ plays}]\!]^{\wedge}))$

(*d*) $[\![\mathfrak{P}(\Delta_{Syd}, \mathcal{A}(\Delta_{sees}, \Delta_{Emily\ plays}))]\!]^{i}$

$= [\![\mathfrak{P}]\!]_{*}([\![\Delta_{Syd}]\!]^{i}, [\![\mathcal{A}]\!]_{+}([\![\Delta_{sees}]\!]^{i}, [\![\Delta_{Emily\ plays}]\!]^{\wedge}))$

(22*a*) Syd sees that Syd sees that Emily plays.

(*b*) $\mathfrak{P}(\Delta_{Syd}, \mathcal{A}(\Delta_{sees}, \mathfrak{P}(\Delta_{Syd}, \mathcal{A}(\Delta_{sees}, \Delta_{Emily\ plays}))))$

(*c*) $[\![\mathfrak{P}(\Delta_{Syd}, \mathcal{A}(\Delta_{sees}, \mathfrak{P}(\Delta_{Syd}, \mathcal{A}(\Delta_{sees}, \Delta_{Emily\ plays}))))]\!]^{\wedge}$

$= [\![\mathfrak{P}]\!]([\![\Delta_{Syd}]\!]^{\wedge}, [\![\mathcal{A}]\!]([\![\mathfrak{P}]\!]([\![\Delta_{Syd}]\!]^{\wedge}, [\![\mathcal{A}]\!]([\![\Delta_{sees}]\!]^{\wedge}, [\![\Delta_{Emily\ plays}]\!]^{\wedge}))))$

(*d*) $[\![\mathfrak{P}(\Delta_{Syd}, \mathcal{A}(\Delta_{sees}, \mathfrak{P}(\Delta_{Syd}, \mathcal{A}(\Delta_{sees}, \Delta_{Emily\ plays}))))]\!]^{i}$

$= [\![\mathfrak{P}]\!]_{*}([\![\Delta_{Syd}]\!]^{i}, [\![\mathcal{A}]\!]_{+}([\![\Delta_{sees}]\!]^{i}, [\![\Delta_{Syd\ sees\ that\ Emily\ plays}]\!]^{\wedge}))$

$= [\![\mathfrak{P}]\!]_{*}([\![\Delta_{Syd}]\!]^{i}, [\![\mathcal{A}]\!]_{+}([\![\Delta_{sees}]\!]^{i}, [\![\mathfrak{P}]\!]([\![\Delta_{Syd}]\!]^{\wedge}, [\![\mathcal{A}]\!]([\![\Delta_{sees}]\!]^{\wedge}, [\![\Delta_{Emily\ plays}]\!]^{\wedge}))))$

$\stackrel{?}{=} [\![\mathfrak{P}]\!]_{*}([\![\Delta_{Syd}]\!]^{i}, [\![\mathcal{A}]\!]_{+}([\![\Delta_{sees}]\!]^{i}, [\![\mathfrak{P}]\!]([\![\Delta_{Syd}]\!]^{\wedge}, [\![\mathcal{A}]\!]^{\wedge}([\![\Delta_{sees}]\!]^{\wedge}, [\![\Delta_{Emily\ plays}]\!]^{\wedge\wedge}))))$

(18*d*) $[\![\mathcal{A}(\Delta_1, \Delta_2)]\!]^{k} = [\![\mathcal{A}]\!]^{k}_{+}([\![\Delta_1]\!]^{k}, [\![\Delta_2]\!]^{k+1})$ Baroque compositionality

(23) *Notation*

- $^{\wedge^{0}}\alpha = \alpha$
- $^{\wedge^{n+1}}\alpha = [^{\wedge}{}^{\wedge^{n}}\alpha]$ $= [^{\wedge^{n}}{}^{\wedge}\alpha]$
- $^{\vee^{0}}\alpha = \alpha$
- $^{\vee^{n+1}}\alpha = [^{\vee}{}^{\vee^{n}}\alpha]$ $= [^{\vee^{n}}{}^{\vee}\alpha]$

- $^{\wedge^{n}_{m}}\alpha = [^{\wedge^{m}}(\lambda \boldsymbol{X}.\, [^{\wedge^{n-m}}\boldsymbol{X}])(\alpha)]$ $0 \leq m \leq n$
- $\mathbf{A}^{0}_{ab} = (\lambda \boldsymbol{f}.\, \lambda \boldsymbol{x}.\, \boldsymbol{f}(\boldsymbol{x}))$ $a, b \in IT, \boldsymbol{f} \in Var_{(ab)}, \boldsymbol{x} \in Var_{a}$
- $\mathbf{A}^{n+1}_{ab} = (\lambda \boldsymbol{f}.\, \lambda \boldsymbol{x}.\, [^{\wedge}\mathbf{A}^{n}_{ab}([^{\vee}\boldsymbol{f}])([^{\vee}\boldsymbol{x}])])$ $a, b \in IT, \boldsymbol{f} \in Var_{(s^{n+1}(ab))}, \boldsymbol{x} \in Var_{(s^{n+1}a)}$

(24) *Baroque Translation*

$|\,Emily\,|^{n} = {}^{\wedge^{n}}\mathbf{e}$

$|\,plays\,|^{n} = {}^{\wedge^{n}}\mathbf{P}$

$|\,Syd\ sees\ that\,|^{n} = {}^{\wedge^{n}}\mathbf{\Sigma}$

$|\,Name\ Pred\,|^{n} = {}^{\wedge^{n}}[^{\vee^{n}}|\,Pred\,|^{n}]([^{\vee^{n}}|\,Name\,|^{n}])$

$|\,Op\ Sent\,|^{n} = {}^{\wedge^{n}}[^{\vee^{n}}|\,Op\,|^{n}]([^{\vee^{n}}|\,Sent\,|^{n+1}])$

(24) *Classical Translation*

$|\,Emily\,| = \mathbf{e}$ $\in Con_e$

$|\,plays\,| = \mathbf{P}$ $\in Con_{(et)}$

$|\,Syd\ sees\ that\,| = \Sigma$ $\in Con_{(st)t}$

$|\,Name\ Pred\,| = |\,Pred\,|(|\,Name\,|)$

$|\,Op\ Sent\,| = |\,Op\,|(^\wedge|\,Sent\,|)$

5. *From Frege to Bäuerle*

(25) George believes that every team player stays in a 5* hotel. Bäuerle (1983)

(26) $\boldsymbol{\lambda i.\, F(i)(\lambda j.\, G(j)(\lambda k.\, H(i,j,k)))}$ Bäuerle constellation

(27) *Contemporary Translation*

$|\,Emily\,|^{n}_{\sim} = {}^{\wedge}{}^{n}_{m}\mathbf{e}$ for some $m \leq n$

$|\,plays\,|^{n}_{\sim} = {}^{\wedge}{}^{n}_{m}\mathbf{P}$ for some $m \leq n$

$|\,Syd\ sees\ that\,|^{n}_{\sim} = {}^{\wedge}{}^{n}_{m}\Sigma$ for some $m \leq n$

$|\,Name\ Pred\,|^{n}_{\sim} = \mathbf{A}^{n}_{et}(|\,Pred\,|^{n}_{\sim})(|\,Name\,|^{n}_{\sim})$

$|\,Op\ Sent\,|^{n}_{\sim} = \mathbf{A}^{n}_{(st)t}(|\,Op\,|^{n}_{\sim})(|\,Sent\,|^{n+1}_{\sim})$

(28) *Definition*
A lexical expression Δ is *trivial* if $m = n$ in the above equation.

(29) *Observations* proofs on request

(a) $[\![\mathbf{A}^{n}_{ab}(\alpha)(\beta)]\!]^{i_0,g}(i_1)\ldots(i_n) = [\![\alpha]\!]^{i_0,g}(i_1)\ldots(i_n)\,([\![\beta]\!]^{i_0,g}(i_1)\ldots(i_n))$ $\alpha \in IL_{(s^n(ab))}, \beta \in IL_{(s^n a)}$

(b) If all lexical expressions are trivial, then:

$$[\![\,|\,\Delta\,|^{n}_{\sim}]\!]^{i,g} = [\![\,{}^{\wedge n}|\,\Delta\,|\,]\!]^{i,g} \;\&\; [\![\,|\,\Delta\,|^{0}_{\sim}]\!]^{i,g} = [\![\,|\,\Delta\,|\,]\!]^{i,g}$$

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