Fregean Compositionality

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1. *Background*

(1*a*) John seeks a unicorn.

(*b*) $\mathfrak{P}(\Delta_{John}, \mathfrak{O}(\Delta_{seeks}, \Delta_{a\ unicorn}))$

(*c*) $\mathfrak{Q}(\Delta_{a\ unicorn}, \Delta_{John\ seeks\ \_\_})$

(2) $\llbracket \mathfrak{F}(\Delta_1,\ldots,\Delta_n)\rrbracket^\wedge = \llbracket \mathfrak{F}\rrbracket(\llbracket \Delta_1\rrbracket^\wedge,\ldots,\llbracket \Delta_n\rrbracket^\wedge)$,

(3) $\llbracket \mathfrak{P}\rrbracket(x,P)(i) = P_i(x_i)$ $\qquad = P(i)(x(i))$

(4) $\llbracket \mathfrak{P}(\Delta_{John},\Delta_{seeks\ a\ unicorn})\rrbracket^i = 1$

iff $\llbracket \mathfrak{P}(\Delta_{John},\Delta_{seeks\ a\ unicorn})\rrbracket^\wedge(i) = 1$ $\qquad$ notation: $\llbracket\Delta\rrbracket^i = \llbracket\Delta\rrbracket^\wedge(i)$

iff $\llbracket \mathfrak{P}\rrbracket(\llbracket\Delta_{John}\rrbracket^\wedge,\llbracket\Delta_{seeks\ a\ unicorn}\rrbracket^\wedge)(i) = 1$ $\qquad$ (2)

iff $\llbracket\Delta_{seeks\ a\ unicorn}\rrbracket^\wedge(i)(\llbracket\Delta_{John}\rrbracket^\wedge(i)) = 1$ $\qquad$ (3)

iff $\llbracket\Delta_{seeks\ a\ unicorn}\rrbracket^i(\llbracket\Delta_{John}\rrbracket^i) = 1$ $\qquad$ notation

(5) $\llbracket \mathfrak{O}\rrbracket(R,Q)(i) = R_i(Q)$ $\qquad$ R of type $(\boldsymbol{s},((\boldsymbol{s},((\boldsymbol{e},\boldsymbol{t}),\boldsymbol{t})),(\boldsymbol{e},\boldsymbol{t})))$, Q of type $(\boldsymbol{s},((\boldsymbol{e},\boldsymbol{t}),\boldsymbol{t}))$, $i\in D_s$

(6) $\llbracket \mathfrak{P}(\Delta_{John},\Delta_{seeks\ a\ unicorn})\rrbracket^i = 1$

iff $\llbracket\Delta_{seeks\ a\ unicorn}\rrbracket^\wedge(i)(\llbracket\Delta_{John}\rrbracket^\wedge(i)) = 1$ $\qquad$ (4)

iff $\llbracket \mathfrak{O}(\Delta_{seeks},\Delta_{a\ unicorn})\rrbracket^\wedge(i)(\llbracket\Delta_{John}\rrbracket^\wedge(i)) = 1$ $\qquad$ def. $\Delta_{seeks\ a\ unicorn}$

iff $\llbracket \mathfrak{O}\rrbracket(\llbracket\Delta_{seeks})\rrbracket^\wedge,\llbracket\Delta_{seeks\ a\ unicorn}\rrbracket^\wedge)(i)(\llbracket\Delta_{John}\rrbracket^\wedge(i)) = 1$ $\qquad$ (2)

iff $\llbracket\Delta_{seeks})\rrbracket^\wedge(i)(\llbracket\Delta_{a\ unicorn}\rrbracket^\wedge)(\llbracket\Delta_{John}\rrbracket^\wedge(i)) = 1$ $\qquad$ (5)

iff $\llbracket\Delta_{seeks}\rrbracket^i(\llbracket\Delta_{John}\rrbracket^i,\llbracket\Delta_{a\ unicorn}\rrbracket^\wedge) = 1$ $\qquad$ notation

2. *Uniform extensionality*

(7) $\llbracket \mathfrak{P}(\Delta_1,\Delta_2)\rrbracket^i = \llbracket\Delta_2\rrbracket^i(\llbracket\Delta_1\rrbracket^i)$

(8) $\llbracket \mathfrak{O}(\Delta_1,\Delta_2)\rrbracket^i = \llbracket\Delta_1\rrbracket^i(\llbracket\Delta_2\rrbracket^\wedge)$

(9) *Definition*

A construction $\mathfrak{F}$ (of n places) is *extensional* iff, at any point $i\in D_s$, the extension of an expression of the form $\mathfrak{F}(\Delta_1,\ldots,\Delta_n)$ at i, is determined by the extensions of its immediate parts at i, i.e.:

- $\llbracket \mathfrak{F}(\Delta_1,\ldots,\Delta_n)\rrbracket^i = \llbracket \mathfrak{F}(\Delta'_1,\ldots,\Delta'_n)\rrbracket^i$ whenever $\llbracket\Delta_1\rrbracket^i = \llbracket\Delta'_1\rrbracket^i,\ldots,\llbracket\Delta_n\rrbracket^i = \llbracket\Delta'_n\rrbracket^i$

for any (appropriate) expressions $\Delta_1, \Delta'_1,\ldots, \Delta_n, \Delta'_n$.

(10) *Definition*

A semantic operation $\llbracket\mathfrak{F}\rrbracket$ is *extensional* iff for any $i\in D_s$:

- $\llbracket \mathfrak{F}\rrbracket(x_1,\ldots,x_n)(i) = \llbracket \mathfrak{F}\rrbracket(x'_1,\ldots,x'_n)(i)$ whenever $x_1(i) = x'_1(i),\ldots, x_n(i) = x'_n(i)$,

for any (appropriate) intensions $x_1, x'_1,\ldots, x_n, x'_n$.

(11) *Facts*

(a) $\mathbb{F}$ is extensional iff for any $i \in D_s$ there is an operation $\llbracket \mathbb{F} \rrbracket^i$ such that

- $\llbracket \mathbb{F}(\Delta_1,\ldots,\Delta_n) \rrbracket^i = \llbracket \mathbb{F} \rrbracket^i(\llbracket \Delta_1 \rrbracket^i,\ldots,\llbracket \Delta_n \rrbracket^i)$,

for any (appropriate) expressions $\Delta_1,\ldots,\Delta_n$.

(b) $\llbracket \mathbb{F} \rrbracket$ is extensional iff for any $i \in D_s$ there is an operation $\llbracket \mathbb{F} \rrbracket^i$ such that

- $\llbracket \mathbb{F} \rrbracket(x_1,\ldots,x_n)(i) = \llbracket \mathbb{F} \rrbracket^i(x_1(i),\ldots,x_n(i))$

for any (appropriate) intensions $x_1,\ldots,x_n$.

(12) *Definition*

A semantic operation $\llbracket \mathbb{F} \rrbracket$ is *uniformly extensional* iff there is an operation $\llbracket \mathbb{F} \rrbracket_*$ such that, for any $i \in D_s$:

- $\llbracket \mathbb{F} \rrbracket(x_1,\ldots,x_n)(i) = \llbracket \mathbb{F} \rrbracket_*(x_1(i),\ldots,x_n(i))$

for any (appropriate) intensions $x_1,\ldots,x_n$.

(13) $$\llbracket \mathbb{F}^{\#} \rrbracket(p,q)(i) = \begin{cases} min(p(i),q(i)), \text{ if } i \neq i^{\#} \\ max(p(i^{\#}),q(i^{\#})), \text{ otherwise} \end{cases}$$

(14) $$\llbracket \mathbb{F}^{\#\#} \rrbracket(\varphi,\psi)(i)(j) = \begin{cases} max(\varphi(i)(j),\psi(i)(j)), \text{ if } \varphi(i)(i) = \psi(i)(i) = 1 \\ min(\varphi(i)(j),\psi(i)(j)), \text{ otherwise} \end{cases}$$

3. *Selective extensionality*

(15*a*) seeks [a French restaurant that serves bouillabaisse]$_1$
(*b*) seeks [a French restaurant that serves ratatouille]$_2$

(16) $$\llbracket \Delta_{(15a)} \rrbracket^i \underset{(11a)}{=} \llbracket \mathcal{O} \rrbracket^i(\llbracket \Delta_{seeks} \rrbracket^i,\llbracket \Delta_1 \rrbracket^i) \underset{(!)}{=} \llbracket \mathcal{O} \rrbracket^i(\llbracket \Delta_{seeks} \rrbracket^i,\llbracket \Delta_2 \rrbracket^i) \underset{(11a)}{=} \llbracket \Delta_{(15b)} \rrbracket^i,$$

(17*a*) thinks [that] Mary is sick
(*b*) thinks [that] 2+2=5

(18) $$\llbracket \Delta_{(17a)} \rrbracket^i \underset{(11a)}{=} \llbracket \mathcal{A} \rrbracket^i(\llbracket \Delta_{thinks} \rrbracket^i,\llbracket \Delta_{Mary\ is\ sick} \rrbracket^i) \underset{(!!)}{=} \llbracket \mathcal{A} \rrbracket^i(\llbracket \Delta_{thinks} \rrbracket^i,\llbracket \Delta_{2+2=5} \rrbracket^i) \underset{(11a)}{=} \llbracket \Delta_{(17b)} \rrbracket^i$$

(19*a*) $\llbracket \mathbb{F}(\Delta_1,\Delta_2) \rrbracket^i = \llbracket \mathbb{F} \rrbracket^i(\llbracket \Delta_1 \rrbracket^i,\llbracket \Delta_2 \rrbracket^i)$

(*b*) $\llbracket \mathbb{F}(\Delta_1,\Delta_2) \rrbracket^i = \llbracket \mathbb{F} \rrbracket_*(\llbracket \Delta_1 \rrbracket^i,\llbracket \Delta_2 \rrbracket^i)$

(*c*) $\llbracket \mathbb{F}(\Delta_1,\Delta_2) \rrbracket^\wedge = \llbracket \mathbb{F} \rrbracket(\llbracket \Delta_1 \rrbracket^\wedge,\llbracket \Delta_2 \rrbracket^\wedge)$

(*c*') $\llbracket \mathbb{F}(\Delta_1,\Delta_2) \rrbracket^i = \llbracket \mathbb{F} \rrbracket_i(\llbracket \Delta_1 \rrbracket^\wedge,\llbracket \Delta_2 \rrbracket^\wedge)$

(20) *Definitions*

(*a*) An *n*-place semantic operation $\llbracket \mathbb{F} \rrbracket$ is *k-extensional* (where $1 \leq k \leq n$) iff for any $i \in D_s$, and any (appropriate) intensions $x_1,\ldots,x_n$, and x'_k: if $x_k(i) = x'_k(i)$, then:

- $\llbracket \mathbb{F} \rrbracket(x_1,\ldots,x_k,\ldots,x_n)(i) = \llbracket \mathbb{F} \rrbracket(x_1,\ldots,x'_k,\ldots,x_n)(i)$.

(*b*) $\llbracket \mathbb{F} \rrbracket$ is *K-extensional* iff $K = \{k \mid k \text{ is } k\text{-extensional}\}$.

(21) *Facts*

(a) $\llbracket \mathfrak{F} \rrbracket$ is k-extensional iff for any $i \in D_s$ there is an operation $\llbracket \mathfrak{F} \rrbracket^i$ such that, for any (appropriate) intensions $x_1, \ldots, x_n$:

- $\llbracket \mathfrak{F} \rrbracket(x_1, \ldots, x_k, \ldots, x_n)(i) = \llbracket \mathfrak{F} \rrbracket^i(x_1, \ldots, x_k(i), \ldots, x_n)$.

(b) $\llbracket \mathfrak{F} \rrbracket$ is K-extensional iff for any $i \in D_s$ there is an operation $\llbracket \mathfrak{F} \rrbracket^i$ such that, for any (appropriate) intensions $x_1, \ldots, x_n$:

- $\llbracket \mathfrak{F} \rrbracket(x_1, \ldots, x_n)(i) = \llbracket \mathfrak{F} \rrbracket^i(x_1[i], \ldots, x_n[i])$,

where (for any $k \leq n$):

- $x_k[i] = \begin{cases} x_k(i), \text{ if } k \in K \\ x_k, \text{ if } k \notin K \end{cases}$

(22) $\llbracket \mathfrak{O} \rrbracket^i(\mathrm{R}, \mathcal{Q}) = \mathrm{R}(\mathcal{Q})$ R of type $((\boldsymbol{s},((\boldsymbol{e},\boldsymbol{t}),\boldsymbol{t})),(\boldsymbol{e},\boldsymbol{t}))$, $\mathcal{Q}$ of type $(\boldsymbol{s},((\boldsymbol{e},\boldsymbol{t}),\boldsymbol{t}))$

(23) $\llbracket \mathfrak{A} \rrbracket(S,p)(i) = S_i(p)$

(24) $\llbracket \mathfrak{A} \rrbracket^i(\mathrm{S},p) = \mathrm{S}(p)$

(25) *Definition*

$\llbracket \mathfrak{F} \rrbracket$ is uniformly K-extensional iff $\llbracket \mathfrak{F} \rrbracket$ is K-extensional and there is an operation $\llbracket \mathfrak{F} \rrbracket_*$ such that, for any $i \in D_s$, $\llbracket \mathfrak{F} \rrbracket_* = \llbracket \mathfrak{F} \rrbracket^i$, as defined in (21$b$).

(26a) $\llbracket \mathfrak{P} \rrbracket_*(\mathrm{x},\mathrm{P}) = \mathrm{P}(\mathrm{x})$

(b) $\llbracket \mathfrak{O} \rrbracket_*(\mathrm{R},\mathcal{Q}) = \mathrm{R}(\mathcal{Q})$

(c) $\llbracket \mathfrak{A} \rrbracket_*(\mathrm{S},p) = \mathrm{S}(p)$

4. *The Hierarchy of Intensions*

(27a) Norman hears that Syd sees that <u>Emily plays</u>.

(b) $\mathfrak{P}(\Delta_{Norman}, \mathfrak{A}(\Delta_{hears}, \mathfrak{P}(\Delta_{Syd}, \mathfrak{A}(\Delta_{sees}, \mathfrak{P}(\Delta_{Emily}, \Delta_{plays})))))$

(c) $\llbracket \mathfrak{P}(\Delta_{Norman}, \mathfrak{A}(\Delta_{hears}, \mathfrak{P}(\Delta_{Syd}, \mathfrak{A}(\Delta_{sees}, \Delta_{Emily\ plays})))) \rrbracket^{\wedge}$

$= \llbracket \mathfrak{P} \rrbracket(\llbracket \Delta_{Norman} \rrbracket^{\wedge}, \llbracket \mathfrak{A} \rrbracket(\llbracket \Delta_{hears} \rrbracket^{\wedge}, \llbracket \mathfrak{P} \rrbracket(\llbracket \Delta_{Syd} \rrbracket^{\wedge}, \llbracket \mathfrak{A} \rrbracket(\llbracket \Delta_{sees} \rrbracket^{\wedge}, \llbracket \Delta_{Emily\ plays} \rrbracket^{\wedge}))))$

$= \llbracket \mathfrak{P} \rrbracket(\llbracket \Delta_{Norman} \rrbracket^{\wedge}, \llbracket \mathfrak{A} \rrbracket(\llbracket \Delta_{hears} \rrbracket^{\wedge}, \llbracket \mathfrak{P} \rrbracket(\llbracket \Delta_{Syd} \rrbracket^{\wedge}, \llbracket \mathfrak{A} \rrbracket(\llbracket \Delta_{sees} \rrbracket^{\wedge}, \llbracket \mathfrak{P} \rrbracket(\llbracket \Delta_{Emily} \rrbracket^{\wedge}, \llbracket \Delta_{plays} \rrbracket^{\wedge})))))$

$= \llbracket \mathfrak{P} \rrbracket(n, \llbracket \mathfrak{A} \rrbracket(H, \llbracket \mathfrak{P} \rrbracket(s, \llbracket \mathfrak{A} \rrbracket(S, \llbracket \mathfrak{P} \rrbracket(e,P)))))$

(d) $\llbracket \mathfrak{P}(\Delta_{Norman}, \mathfrak{A}(\Delta_{hears}, \mathfrak{P}(\Delta_{Syd}, \mathfrak{A}(\Delta_{sees}, \mathfrak{P}(\Delta_{Emily}, \Delta_{plays}))))) \rrbracket^i$

$= \llbracket \mathfrak{P} \rrbracket_*(\llbracket \Delta_{Norman} \rrbracket^i, \llbracket \mathfrak{A} \rrbracket_*(\llbracket \Delta_{hears} \rrbracket^i, \llbracket \Delta_{Syd\ sees\ that\ Emily\ plays} \rrbracket^{\wedge}))$

$= \llbracket \mathfrak{P} \rrbracket_*(\llbracket \Delta_{Norman} \rrbracket^i, \llbracket \mathfrak{A} \rrbracket_*(\llbracket \Delta_{hears} \rrbracket^i, \llbracket \mathfrak{P} \rrbracket(\llbracket \Delta_{Syd} \rrbracket^{\wedge}, \llbracket \mathfrak{A} \rrbracket(\llbracket \Delta_{sees} \rrbracket^{\wedge}, \llbracket \mathfrak{P} \rrbracket(\llbracket \Delta_{Emily} \rrbracket^{\wedge}, \llbracket \Delta_{plays} \rrbracket^{\wedge})))))$

$= \llbracket \mathfrak{P} \rrbracket_*(\mathrm{n}, \llbracket \mathfrak{A} \rrbracket_*(H_i, \llbracket \mathfrak{P} \rrbracket(s, \llbracket \mathfrak{A} \rrbracket(S, \llbracket \mathfrak{P} \rrbracket(e,P)))))$

(e) $\llbracket \mathfrak{P}(\Delta_{Norman}, \mathfrak{A}(\Delta_{hears}, \mathfrak{P}(\Delta_{Syd}, \mathfrak{A}(\Delta_{sees}, \mathfrak{P}(\Delta_{Emily}, \Delta_{plays}))))) \rrbracket^i$

$= \llbracket \mathfrak{P} \rrbracket_*(\mathrm{n}, \llbracket \mathfrak{A} \rrbracket_*(H_i, \lambda j.\ \llbracket \mathfrak{P} \rrbracket_*(\mathrm{s}, \llbracket \mathfrak{A} \rrbracket_*(S_j, \lambda k.\ \llbracket \mathfrak{P} \rrbracket_*(\mathrm{e}, P_k)))))$

$= H_i(\mathrm{n}, \lambda j.\ S_j(\mathrm{s}, \lambda k.\ P_k(\mathrm{e})))$

(28) $[\![\mathfrak{P}(\Delta_{Norman}, \mathcal{A}(\Delta_{hears}, \mathfrak{P}(\Delta_{Syd}, \mathcal{A}(\Delta_{sees}, \mathfrak{P}(\Delta_{Emily}, \Delta_{plays})))))]\!]^{i}$

= $[\![\mathfrak{P}]\!]_{*}([\![\Delta_{Norman}]\!]^{i}, [\![\mathcal{A}]\!]_{*}([\![\Delta_{hears}]\!]^{i},$
$[\![\mathfrak{P}]\!]_{**}([\![\Delta_{Syd}]\!]^{\wedge}, [\![\mathcal{A}]\!]_{**}([\![\Delta_{sees}]\!]^{\wedge}, [\![\mathfrak{P}]\!]_{***}([\![\Delta_{Emily}]\!]^{\wedge\wedge}, [\![\Delta_{plays}]\!]^{\wedge\wedge})))))$

= $[\![\mathfrak{P}]\!]_{*}(\mathrm{n}, [\![\mathcal{A}]\!]_{*}(H_i, [\![\mathfrak{P}]\!]_{**}(s, [\![\mathcal{A}]\!]_{**}(S, [\![\mathfrak{P}]\!]_{***}([\![\Delta_{Emily}]\!]^{\wedge\wedge}, [\![\Delta_{plays}]\!]^{\wedge\wedge})))))$

(29) *Definition*
If Δ is any expression, then:

- $[\![\Delta]\!]^{\wedge^{1}} := [\![\Delta]\!]^{\wedge}$
- $[\![\Delta]\!]^{\wedge^{n+1}} := \lambda i \in D_s.\ [\![\Delta]\!]^{\wedge}$

(30*a*) $[\![\mathfrak{P}]\!]_{**}(x,P) = \lambda j \in D_s.\ [\![\mathfrak{P}]\!]_{*}(x_j,P_j)$ $\qquad x \in D_{se}, P \in D_{s(et)}$

(*b*) $[\![\mathcal{A}]\!]_{**}(A,\varphi) = \lambda j \in D_s.\ [\![\mathcal{A}]\!]_{*}(A_j,\varphi_j)$ $\qquad A \in D_{s((st)(et))}, \varphi \in D_{s(st)}$

(*c*) $[\![\mathfrak{P}]\!]_{***}(\boldsymbol{x},\boldsymbol{P}) = \lambda j \in D_s.\ [\![\mathfrak{P}]\!]_{**}(\boldsymbol{x}_j,\boldsymbol{P}_j)$ $\qquad \boldsymbol{x} \in D_{s(se)}, \boldsymbol{P} \in D_{(s(s(et)))}$

5. *From Frege to Bäuerle*

(31) Syd sees that every band member is drinking.

(32) $[\![\mathfrak{P}]\!]_{*}(\mathrm{s},[\![\mathcal{A}]\!]_{*}(S_i, \lambda j.\ [\![\mathfrak{S}]\!]_{*}([\![\mathfrak{D}]\!]_{*}(\forall,B_i),D_j)))$

= $S_i(\mathrm{s}, \lambda j.\ \forall(B_i,D_j))$ $\qquad$ [= $S_i(\mathrm{s}, \lambda j.\ \forall(B_i)(D_j))$, by notational convention]

(33*a*) $[\![\mathfrak{S}]\!](Q,P)(i) = Q_i(P_i)$ $\qquad Q \in D_{(s((et)t)))}, P \in D_{(s(et))}$

(*b*) $[\![\mathfrak{D}]\!](D,P)(i) = D_i(P_i)$ $\qquad D \in D_{(s((et)((et)t)))}, P \in D_{(s(et))}$

(34*a*) $[\![\mathfrak{P}]\!]_{*}(\mathrm{s},[\![\mathcal{A}]\!]_{*}(S_i, \lambda j.\ [\![\mathfrak{S}]\!]_{*}([\![\mathfrak{D}]\!]_{*}(\forall,B_j),D_j)))$

= $S_i(\mathrm{s}, \lambda j.\ \forall(B_j,D_j))$

(*b*) $[\![\mathfrak{P}]\!]_{*}(\mathrm{s},[\![\mathcal{A}]\!]_{*}(S_i, \lambda j.\ [\![\mathfrak{Q}]\!]_{*}([\![\mathfrak{D}]\!]_{*}(\forall,B_j),(\lambda \mathrm{x}.\ [\![\mathfrak{P}]\!]_{*}(\mathrm{x},D_j)))))$

= $S_i(\mathrm{s}, \lambda j.\ \forall(B_i, \lambda x.\ D_j(x)))$

(*c*) $[\![\mathfrak{Q}]\!]_{*}([\![\mathfrak{D}]\!]_{*}(\forall,B_i),(\lambda \mathrm{x}.[\![\mathfrak{P}]\!]_{*}(\mathrm{s},[\![\mathcal{A}]\!]_{*}(S_i, \lambda j.\ [\![\mathfrak{P}]\!]_{*}(\mathrm{x},D_j)))))$

= $\forall(B_i, \lambda x.S_i(\mathrm{s}, \lambda j.\ D_j(x)))$

(35) $[\![\mathfrak{P}]\!]_{*}([\![\Delta_{Syd}]\!]^{i},[\![\mathcal{A}]\!]_{*}([\![\Delta_{sees}]\!]^{i}([\![\mathfrak{S}]\!]_{**}([\![\mathfrak{D}]\!]_{**}([\![\Delta_{every}]\!]^{\wedge},[\![\Delta_{band\ member}]\!]^{\wedge}),[\![\Delta_{is\ drinking}]\!]^{\wedge}))))$

= $[\![\mathfrak{P}]\!]_{*}([\![\Delta_{Syd}]\!]^{i},[\![\mathcal{A}]\!]_{*}([\![\Delta_{sees}]\!]^{i}([\![\mathfrak{S}]\!]([\![\mathfrak{D}]\!]([\![\Delta_{every}]\!]^{\wedge},[\![\Delta_{band\ member}]\!]^{\wedge}),[\![\Delta_{is\ drinking}]\!]^{\wedge}))))$

= $S_i(\mathrm{s}, \lambda j.\ \forall(B_i, \lambda x.\ D_j(x)))$

(36) Norman hears that Syd sees that every band member is drinking.

(37) $[\![\mathfrak{P}]\!]_{*}(\mathrm{n},[\![\mathcal{A}]\!]_{*}(H_i, \lambda j.\ [\![\mathfrak{P}]\!]_{*}(\mathrm{s},[\![\mathcal{A}]\!]_{*}(S_j, \lambda k.\ [\![\mathfrak{S}]\!]_{*}([\![\mathfrak{D}]\!]_{*}(\forall,B_j),D_k)))))$

= $H_i(\mathrm{n}, \lambda j.\ S_j(\mathrm{s}, \lambda k.\ \forall(B_j,D_k)))$

(38) $H_i(\mathrm{n}, \lambda j.\ S_j(\mathrm{s}, \lambda k.\ \forall(B_k,D_k)))$

(39) $[\![\mathfrak{P}(\Delta_{Norman},\mathcal{A}(\Delta_{hears},\mathfrak{P}(\Delta_{Syd},\mathcal{A}(\Delta_{sees},\mathbb{S}(\mathfrak{D}(\Delta_{every},\Delta_{band\ member}),\Delta_{is\ drinking})))))]\!]^{i}$

$= [\![\mathfrak{P}]\!]_{*}([\![\Delta_{Norman}]\!]^{i},[\![\mathcal{A}]\!]_{*}([\![\Delta_{hears}]\!]^{i},$
$[\![\mathfrak{P}]\!]_{**}([\![\Delta_{Syd}]\!]^{\wedge},[\![\mathcal{A}]\!]_{**}([\![\Delta_{sees}]\!]^{\wedge},$
$[\![\mathbb{S}]\!]_{***}([\![\mathfrak{D}]\!]_{***}([\![\Delta_{every}]\!]^{\wedge\wedge},[\![\Delta_{band\ member}]\!]^{\wedge\wedge}),[\![\Delta_{is\ drinking}]\!]^{\wedge\wedge})))))$

$= H_{i}(\mathrm{n}, \lambda j.\ S_{j}(\mathrm{s}, [\![\mathbb{S}]\!]_{***}([\![\mathfrak{D}]\!]_{***}([\lambda j.\lambda k.\ \forall], [\lambda j.\lambda k.\ B_{k}]), [\lambda j.\lambda k.\ D_{k}]))\ (j)\ ))$

$= H_{i}(\mathrm{n}, \lambda j.\ S_{j}(\mathrm{s}, \lambda k.\ \forall(B_{k},D_{k})))$

(40*a*) $[\![\mathbb{S}]\!]_{***}(\boldsymbol{Q},\boldsymbol{P}) = \lambda k \in D_{s}.\ [\![\mathbb{S}]\!]_{**}(\boldsymbol{Q}_{k},\boldsymbol{P}_{k})$ $\quad \boldsymbol{Q}{\in}D_{(\boldsymbol{s}(\boldsymbol{s}((\boldsymbol{et})\boldsymbol{t})))}, \boldsymbol{P}{\in}D_{(\boldsymbol{s}(\boldsymbol{s}(\boldsymbol{et}))}$

(*b*) $[\![\mathfrak{D}]\!]_{***}(\boldsymbol{D},\boldsymbol{P}) = \lambda k \in D_{s}.\ [\![\mathfrak{D}]\!]_{**}(\boldsymbol{D}_{k},\boldsymbol{P}_{k})$ $\quad \boldsymbol{D}{\in}D_{(\boldsymbol{s}(\boldsymbol{s}((\boldsymbol{et})((\boldsymbol{et})\boldsymbol{t}))))}, \boldsymbol{P}{\in}D_{(\boldsymbol{s}(\boldsymbol{s}(\boldsymbol{et})))}$

(41) $[\lambda j.\ \lambda k.\ \alpha]$

(42*a*) $[\lambda j.\lambda k.\ B_{k}]$

(*b*) $[\lambda j.\lambda k.\ B_{j}]$

(43) Roger thinks that Norman hears that Syd sees that every band member is drinking.

(44*a*) $\ldots [\lambda i_{m}.\ \ldots [\![\mathfrak{D}]\!]_{*}([\![\Delta_{\delta}]\!](i_{n}))([\![\Delta_{\rho}]\!](i_{n}))\ \ldots]$

(*b*) $\ldots [\lambda i_{m}.\ \ldots [\![\mathfrak{D}]\!]_{*}([\![\Delta_{\delta}]\!](i_{n}))([\![\Delta_{\rho}]\!](i_{m}))\ \ldots]$

(45) $\ldots [\ldots [\![\mathfrak{D}]\!]_{*\ldots}(\ [\![\Delta_{\delta}]\!]^{\wedge\ldots})([\![\Delta_{\rho}]\!]^{\wedge\ldots})\ \ldots]$

(46*a*) $[\![\Delta_{\rho}]\!]^{\wedge\ldots} = \lambda i_{1}.\ \ldots\ \lambda i_{m}.\ \ldots\ \lambda i_{n}.\ [\![\Delta_{\rho}]\!](i_{n})$

(*b*) $[\![\Delta_{\rho}]\!]_{m}^{\wedge\ldots} = \lambda i_{1}.\ \ldots\ \lambda i_{m}.\ \ldots\ \lambda i_{n}.\ [\![\Delta_{\rho}]\!](i_{m})$

(47) Syd said that every brother of Emily's <u>is a band member</u>.

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Appendix: IL-implementation

a) Syntax of fragment

The fragment contains the key examples in the main text. The lexicon contains the following sets of expressions:

- *Names*: Δ_{Emily}, Δ_{Norman} , Δ_{Syd} ,...
- *Predicates* = Δ_{plays}, $\Delta_{is\ drinking}$ ...
- *Attitude Verbs*: Δ_{sees}, Δ_{hears}, ...
- *Nouns*: $\Delta_{band\ member}$, ...
- *Determiners*: Δ_{every}, Δ_{a} , ...

The syntax covers the constructions discussed above and contains the following rules:

R1 If Δ_{NN} is a *Name* and Δ_P is a predicate, then $\mathfrak{P}(\Delta_{NN}, \Delta_P)$ is a *Sentence*.

R2 If Δ_A is an *Attitude Verb* and Δ_S is a *Sentence*, then $\mathfrak{A}(\Delta_A, \Delta_S)$ is a *Predicate*.

R3 If Δ_D is a *Determiner* and Δ_N is a *Noun*, then $\mathfrak{D}(\Delta_D, \Delta_N)$ is a *Quantifier*.

R4 If Δ_Q is a *Quantifier* and Δ_P is a *Predicate*, then $\mathfrak{Q}(\Delta_Q, \Delta_P)$ is a *Sentence*.

b) *IL*: definitions and notation

(i) *Types of indirect intensions*

The interpretation of the fragment will proceed indirectly, by way of a compositional interpretation into Montague's (1970) language *IL* of intensional type logic. The language is based on infinite sets Var_a of variables of any type a and unspecified sets Con_a of (non-logical) constants of type a, and consists of a set IL_a of terms of (any) type a:

- $Var_a \subseteq IL_a$.
- $Con_a \subseteq IL_a$.
- If $\alpha \in IL_{ab}$ and $\beta \in IL_a$, then $\alpha(\beta) \in IL_b$.
- If $\boldsymbol{x} \in IL_a$ and $\alpha \in IL_b$, then $(\boldsymbol{\lambda x.}\ \alpha) \in IL_{ab}$.
- If $\alpha \in IL_a$ and $\beta \in IL_a$, then $(\alpha\boldsymbol{=}\beta) \in IL_{\boldsymbol{t}}$.
- If $\alpha \in IL_{sa}$, then $[^\vee\alpha] \in IL_a$.
- If $\alpha \in IL_a$, then $[^\wedge\alpha] \in IL_{sa}$.

Following Montague (1970), logical constants and operators (like $\vee$, $\wedge$, $\exists$ and $\forall$) may be taken as abbreviations. *IL*-terms receive their denotations relative to models $\mathcal{M} = (\boldsymbol{D_e}, \boldsymbol{D_s}, \mathbf{F})$, indices $i \in \boldsymbol{D_s}$, and $\mathcal{M}$-assignments g:

- $[\![\boldsymbol{x}]\!]^{\mathcal{M}, i, g} = g(\boldsymbol{x})$ if $\boldsymbol{x} \in Var_a$.
- $[\![\mathbf{c}]\!]^{\mathcal{M}, i, g} = \mathbf{F}(\mathbf{c})(i)$ if $\mathbf{c} \in Con_a$.
- $[\![\alpha(\beta)]\!]^{\mathcal{M}, i, g} = [\![\alpha]\!]^{\mathcal{M}, i, g}([\![\beta]\!]^{\mathcal{M}, i, g})$.
- $[\![(\lambda \boldsymbol{x}^a.\ \alpha)]\!]^{\mathcal{M}, i, g} = \{(u, [\![\alpha]\!]^{\mathcal{M}, i, g[x/u]}) \mid u \in D_a\}$.
- $[\![(\alpha = \beta)]\!]^{\mathcal{M}, i, g} = \{1 \mid [\![\alpha]\!]^{\mathcal{M}, i, g} = [\![\beta]\!]^{\mathcal{M}, i, g}\}$.
- $[\![[^\vee\alpha]]\!]^{\mathcal{M}, i, g} = [\![\alpha]\!]^{\mathcal{M}, i, g}(i)$.
- $[\![[^\wedge\alpha]]\!]^{\mathcal{M}, i, g} = \{(j, [\![\alpha]\!]^{\mathcal{M}, j, g}) \mid j \in D_s\}$.

Two *IL*-terms α and β of the same type are *logically equivalent* iff $[\![\alpha]\!]^{\mathcal{M}, i, g} = [\![\beta]\!]^{\mathcal{M}, i, g}$, for all models $\mathcal{M}$, indices i, and assignments g; notation: $\alpha \equiv \beta$.

(ii) *Iteration of IL-operators*

The indirect intensions in the hierarchy (29) are of the types of the form ($\boldsymbol{s}^n$a):

- $(\boldsymbol{s}^0 a) = a$

- $(s^{n+1}a) = (s(s^n a))$

For each *IL*-term α the term $[{}^{\wedge^n}\alpha]$ denotes its n^{th} indirect intension:

- $[{}^{\wedge^0}\alpha] = \alpha$
- $[{}^{\wedge^{n+1}}\alpha] = [{}^{\wedge}{}^{\wedge^n}\alpha]$ $\quad (= [{}^{\wedge^n}{}^{\wedge}\alpha])$

The indirect interpretation algorithms will also make use of iterated index application $[{}^{\vee^n}\alpha]$:

- $[{}^{\vee^{n+1}}\alpha] = [{}^{\vee^0}\alpha] = \alpha$
- $[[{}^{\vee^{n+1}}\alpha] = [{}^{\vee}{}^{\vee^n}\alpha]$ $\quad (= [{}^{\vee^n}{}^{\vee}\alpha])$

For each *IL*-term α and any $n \geq 0$ the term $[{}^{\wedge^n_m}\alpha]$ designates the twisted versions of its n^{th} indirect intension:

- ${}^{\wedge^n_m}\alpha = [{}^{\wedge^m}(\lambda X.\,[{}^{\wedge^{n-m}}X])(\alpha)]$ $\quad 0 \leq m \leq n$

Functional application is defined recursively on the hierarchy of indirect intensions and twisted senses:

- $\mathbf{A}^0_{ab} = (\lambda f.\,\lambda x.\,f(x))$ $\quad f \in Var_{(ab)},\ x \in Var_a$
- $\mathbf{A}^{n+1}_{ab} = (\lambda f.\,\lambda x.\,[{}^{\wedge}\mathbf{A}^n_{ab}([{}^{\vee}f])([{}^{\vee}x])])$ $\quad f \in Var_{(s^{n+1}(ab))},\ x \in Var_{(s^{n+1}a)}$

c) *Indirect interpretation*

(i) *Standard translation*

For each expression Δ from the fragment defined in ***a)*** the *IL*-term $|\alpha|$ denotes its extension:

- $\{|\Delta_{Emily}|, |\Delta_{Norman}|, |\Delta_{Syd}|, \ldots\} = \{\mathbf{e}, \mathbf{n}, \mathbf{s}, \ldots\} \subseteq Con_{\boldsymbol{e}}$
- $\{|\Delta_{plays}|, \Delta_{is\ drinking}|, \ldots\} = \{\mathbf{P}, \mathbf{D}, \ldots\} \subseteq Con_{(\boldsymbol{et})}$
- $\{|\Delta_{sees}|, |\Delta_{hears}|, \ldots\} = \{\mathbf{S}, \mathbf{H}, \ldots\} \subseteq Con_{((\boldsymbol{st})\ \boldsymbol{et}))}$
- $\{|\Delta_{band\ member}|, \ldots\} = \{\mathbf{B}, \ldots\} \subseteq Con_{(\boldsymbol{et})}$
- $|\Delta_{every}| = [\lambda P^{e\,t}.\,\lambda Q^{e\,t}.\,(\forall x^e)\,[P(x) \rightarrow Q(x)]$ $\quad =: \mathbf{ALL}$
- $|\Delta_a| = [\lambda P^{e\,t}.\,\lambda Q^{e\,t}.\,(\exists x^e)\,[P(x)\,) \wedge Q(x)]$

S1 $\quad |\mathfrak{P}(\Delta_{NN}, \Delta_P)| = |\Delta_P|\,(|\Delta_{NN}|)$

S2 $\quad |\mathfrak{A}(\Delta_A, \Delta_S)| = |\Delta_A|\,([{}^{\wedge}|\Delta_S|])$

S3 $\quad |\mathfrak{D}(\Delta_D, \Delta_N)| = |\Delta_D|\,(|\Delta_N|)$

S4 $\quad |\mathfrak{Q}(\Delta_Q, \Delta_P)| = |\Delta_Q|\,(|\Delta_P|)$

(ii) *Baroque translation*

For each expression Δ from the fragment defined in ***a)*** the *IL*-term $|\alpha|^n$ denotes its extension n^{th} indirect intension, which coincides with its extension if $n = 0$. In opaque positions the translation increases the level of indirectness :

- $|\Delta|^n = {}^{\wedge^n}|\Delta|$ if Δ is lexical

B1 $\quad |\mathfrak{P}(\Delta_{NN}, \Delta_P)|^n = \mathbf{A}^n_{\boldsymbol{et}}\,(|\Delta_P|^n)\,(|\Delta_{NN}|^n)$

B2 $\quad |\mathfrak{A}(\Delta_A, \Delta_S)|^n = \mathbf{A}^n_{(\boldsymbol{st})(\boldsymbol{et})}\,(|\Delta_A|^n)\,(|\Delta_S|^{n+1})$

B3 $\quad |\mathfrak{D}(\Delta_D, \Delta_N)|^n = \mathbf{A}^n_{(\boldsymbol{et})((\boldsymbol{et})\boldsymbol{t})}\,(|\Delta_D|^n)\,(|\Delta_N|^n)$

B4 $\quad |\mathfrak{Q}(\Delta_Q, \Delta_P)|^n = \mathbf{A}^n_{(\boldsymbol{et})\boldsymbol{t}}\,(|\Delta_Q|^n)\,(|\Delta_P|^n)$

(iii) *Underspecified translation*

For each expression Δ from the fragment defined in ***a)*** the set $|\alpha|_{\sim}^{n}$ of *IL*-terms contains all possible choices of n^{th} indirect twisted senses in flexible argument positions; in inflexible positions the semantic operations are distributed, flexible positions bring in twisted senses. The technique is the same as in Rooth's (1985) alternative semantics of focus:

- $|\Delta|_{\sim}^{n} = \{|\Delta|^{n}\}$, if Δ is lexical

U1 $|\mathfrak{P}(\Delta_n, \Delta_P)|_{\sim}^{n} = \{\, \mathbf{A}^{n}_{et}(\alpha)(\beta) \mid \alpha \in |\Delta_P|_{\sim}^{n}, \beta \in |\Delta_n|_{\sim}^{n}\}$

U2 $|\mathfrak{A}(\Delta_A, \Delta_S)|_{\sim}^{n} = \{\, \mathbf{A}^{n}_{(st)(et)}(\alpha)(\beta) \mid \alpha \in |\Delta_A|_{\sim}^{n}, \beta \in |\Delta_S|_{\sim}^{n+1}\}$

U3 $|\mathfrak{D}(\Delta_D, \Delta_N)|_{\sim}^{n} = \{\mathbf{A}^{n}_{(et)((et)t)}(\alpha)(^{\wedge}{}^{n}_{m}{}^{\vee}{}^{n}\beta) \mid m \leq n, \alpha \in |\Delta_D|_{\sim}^{n}, \beta \in |\Delta_N|_{\sim}^{n}\}$

U4 $|\mathfrak{Q}(\Delta_Q, \Delta_P)|_{\sim}^{n} = \{\, \mathbf{A}^{n}_{(et)t)}(\alpha)(\beta) \mid \alpha \in |\Delta_Q|_{\sim}^{n}, \beta \in |\Delta_P|_{\sim}^{n}\}$

d) *Comparison*
(i) *Principal observations*
Let Δ be any expression in the above fragment. Then:

(P1) $|\Delta|^{0} \equiv |\Delta|$

(P2) $|\Delta| \equiv \alpha \in |\Delta|_{\sim}^{0}$, for some *IL*-term α

(ii) *Auxiliary observations*

For all *IL*-terms α, $n \geq m \geq 0$, *IL*-models $\mathcal{M} = (D_e, D_s, \mathbf{F})$, $i_0, \ldots, i_{n+1} \in D_s$, and $\mathcal{M}$-assignments g the following hold:

(A1) $\llbracket {}^{\vee}{}^{n}{}^{\wedge}{}^{n}\alpha \rrbracket^{\mathcal{M}, i_0, g} = \llbracket \alpha \rrbracket^{\mathcal{M}, i_0, g}$

(A2) $\llbracket {}^{\wedge}{}^{n}\alpha \rrbracket^{\mathcal{M}, i_0, g}(i_1)\ldots(i_n) = \llbracket \alpha \rrbracket^{\mathcal{M}, i_n, g}$

(A3) $\llbracket {}^{\wedge}{}^{n+1}\alpha \rrbracket^{\mathcal{M}, i_0, g}(i_1)\ldots(i_n) = \llbracket {}^{\wedge}\alpha \rrbracket^{\mathcal{M}, i_{n+1}, g}$

(A4) $\llbracket \mathbf{A}^{n}_{ab}({}^{\wedge}{}^{n}\alpha)({}^{\wedge}{}^{n}\beta) \rrbracket^{\mathcal{M}, i_0, g} = \llbracket {}^{\wedge}{}^{n}\alpha(\beta) \rrbracket^{\mathcal{M}, i_0, g}$ where, for some types a and b, $\alpha \in IL_{(s^{n}(ab))}$, $\beta \in IL_{(s^{n}a)}$

(A5) $|\Delta|^{n} \equiv [{}^{\wedge}{}^{n}|\Delta|]$ ⇒ *(P1)*

(A6) $\llbracket {}^{\wedge}{}^{n}_{m}\alpha \rrbracket^{\mathcal{M}, i_0, g}(i_1)\ldots(i_n) = \llbracket \alpha \rrbracket^{\mathcal{M}, i_m, g}$

(A7) $\llbracket {}^{\wedge}{}^{n}_{n}\alpha \rrbracket^{\mathcal{M}, i_0, g} = \llbracket {}^{\wedge}{}^{n}\alpha \rrbracket^{\mathcal{M}, i_0, g}$

(A8) $\llbracket \mathbf{A}^{n}_{ab}(\alpha)(\beta) \rrbracket^{\mathcal{M}, i_0, g}(i_1)\ldots(i_n) = \llbracket \alpha \rrbracket^{\mathcal{M}, i_0, g}(i_1)\ldots(i_n)(\llbracket \beta \rrbracket^{\mathcal{M}, i_0, g}(i_1)\ldots(i_n))$

where α and β are as in *(d)*

(A9) $[{}^{\wedge}{}^{n}|\Delta|] \equiv \alpha \in |\Delta|_{\sim}^{n}$, for some *IL*-term α ⇒ *(P2)*

e) *Example*

(36) Norman hears that Syd sees that every band member is drinking.

- *Underlying structure:*
  $\mathfrak{P}(\Delta_{Norman}, \mathfrak{A}(\Delta_{hears}, \mathfrak{P}(\Delta_{Syd}, \mathfrak{A}(\Delta_{sees}, \mathfrak{Q}(\mathfrak{D}(\Delta_{every}, \Delta_{band\ member}), \Delta_{is\ drinking})))))$

(i) *Standard translation*

• $\big|\, \mathcal{Q}(\mathcal{D}(\Delta_{every}, \Delta_{band\ member}), \Delta_{is\ \ drinking}) \,\big|$

= $\big|\, \Delta_{every} \,\big| (\big|\, \Delta_{band\ member} \,\big|)\ (\big|\, \Delta_{is\ \ drinking} \,\big|)$

= $\mathbf{ALL(B)\ (D)}$

≡ $\mathbf{(\forall x^{e})\ [B(x) \rightarrow D(x)]}$

• $\big|\, \mathcal{P}(\Delta_{Syd}, \mathcal{A}(\Delta_{sees}, \mathcal{Q}(\mathcal{D}(\Delta_{every}, \Delta_{band\ member}), \Delta_{is\ \ drinking}))) \,\big|$

= $\mathbf{S(s, [{}^{\wedge}} \,\big|\, \mathcal{Q}(\mathcal{D}(\Delta_{every}, \Delta_{band\ member}), \Delta_{is\ \ drinking}) \,\big|\mathbf{])}$

≡ $\mathbf{S(s, [{}^{\wedge}\ (\forall x^{e})\ [B(x) \rightarrow D(x)]\ ])}$

• $\big|\, (36) \,\big|$

= $\mathbf{H(n, [{}^{\wedge}} \,\big|\, \mathcal{P}(\Delta_{Syd}, \mathcal{A}(\Delta_{sees}, \mathcal{Q}(\mathcal{D}(\Delta_{every}, \Delta_{band\ member}), \Delta_{is\ \ drinking}))) \,\big|\mathbf{])}$

≡ $\mathbf{H(n, [{}^{\wedge}\ S(s, [{}^{\wedge}\ (\forall x^{e})\ [B(x) \rightarrow D(x)]\ ]])}$

⇒ $[\![\, |\, (36)\, |\,]\!]^{\mathbb{M}, i_0, g} = H_i(\mathrm{n}, \lambda j.\ S_j(\lambda k.\ T_k \subseteq D_k))(\mathrm{s}))$ using notational conventions from the text

(ii) *Baroque translation*

• $\big|\ \mathcal{P}(\Delta_{Norman}, \mathcal{A}(\Delta_{hears}, \mathcal{P}(\Delta_{Syd}, \mathcal{A}(\Delta_{sees}, \mathcal{Q}(\mathcal{D}(\Delta_{every}, \Delta_{team\ member}), \Delta_{is\ drinking}))))) \ \big|^{0}$

= $\Big[\!\!\Big[\mathbf{A}^{0}_{et}(\mathbf{A}^{0}_{(st)(et)}(\mathbf{H},\mathbf{A}^{1}_{et}(\mathbf{A}^{1}_{(st)(et)}({}^{\wedge}\mathbf{S},\mathbf{A}^{2}_{((et)t}(\mathbf{A}^{2}_{(et)((et)t)}({}^{\wedge\wedge}\mathbf{ALL},{}^{\wedge\wedge}\mathbf{T}),{}^{\wedge\wedge}\mathbf{D})),{}^{\wedge}\mathbf{s})),\mathbf{n})\Big]\!\!\Big]^{\mathbb{M},i,g}$

= $[\![\mathbf{H}]\!]^{\mathbb{M},i,g}([\![\mathbf{n}]\!]^{\mathbb{M},i,g}, \lambda j.\ [\![\mathbf{S}]\!]^{\mathbb{M},j,g}(\lambda k.\ [\![{}^{\wedge\wedge}\mathbf{ALL}]\!]^{\mathbb{M},i,g}(j)(k)([\![{}^{\wedge\wedge}\mathbf{T}]\!]^{\mathbb{M},i,g}(j)(k))([\![{}^{\wedge\wedge}\mathbf{D}]\!]^{\mathbb{M},j,g})(j)(k))([\![\mathbf{s}]\!]^{\mathbb{M},j,g}))$

= $[\![\mathbf{H}]\!]^{\mathbb{M},i,g}([\![\mathbf{n}]\!]^{\mathbb{M},i,g}, \lambda j.\ [\![\mathbf{S}]\!]^{\mathbb{M},j,g}(\lambda k.\ [\![\mathbf{ALL}]\!]^{\mathbb{M},k,g}([\![\mathbf{T}]\!]^{\mathbb{M},k,g})([\![\mathbf{D}]\!]^{\mathbb{M},k,g}))([\![\mathbf{s}]\!]^{\mathbb{M},j,g}))$

= $H_i(\mathrm{n}, \lambda j.\ S_j(\lambda k.\ T_k \subseteq D_k))(\mathrm{s}))$ $= [\![\, |\, (36)\, |\,]\!]^{\mathbb{M}, i_0, g}$

(iii) *Underspecified translation*

• $\big|\ \mathcal{P}(\Delta_{Norman}, \mathcal{A}(\Delta_{hears}, \mathcal{P}(\Delta_{Syd}, \mathcal{A}(\Delta_{sees}, \mathcal{Q}(\mathcal{D}(\Delta_{every}, \Delta_{team\ member}), \Delta_{is\ drinking}))))) \ \big|^{0}_{\sim}$

= $\{\mathbf{A}^{0}_{et}(\mathbf{A}^{0}_{(st)(et)}(\alpha,\mathbf{A}^{1}_{et}(\mathbf{A}^{1}_{(st)(et)}(\beta,\mathbf{A}^{2}_{((et)t}(\mathbf{A}^{2}_{(et)((et)t)}(\gamma,{}^{\wedge}{}^{2}_{m}{}^{\vee}{}^{2}\delta),\varepsilon)),\sigma)),\nu) \mid m \le 2,$
$\alpha \in \big|\, \Delta_{hears} \,\big|^{0}_{\sim}, \alpha \in \big|\, \Delta_{sees} \,\big|^{1}_{\sim}, \gamma \in \big|\, \Delta_{every} \,\big|^{2}_{\sim}, \delta \in \big|\, \Delta_{team\ member} \,\big|^{2}_{\sim},$
$\varepsilon \in \big|\, \Delta_{is\ drinking} \,\big|^{2}_{\sim}, \sigma \in \big|\, \Delta_{Syd} \,\big|^{1}_{\sim}, \nu \in \big|\, \Delta_{Norman} \,\big|^{0}_{\sim}\}$

= $\{\mathbf{A}^{0}_{et}(\mathbf{A}^{0}_{(st)(et)}(\mathbf{H},\mathbf{A}^{1}_{et}(\mathbf{A}^{1}_{(st)(et)}({}^{\wedge}\mathbf{S},\mathbf{A}^{2}_{((et)t}(\mathbf{A}^{2}_{(et)((et)t)}({}^{\wedge\wedge}\mathbf{ALL},{}^{\wedge}{}^{2}_{0}{}^{\vee}{}^{2}\,{}^{\wedge\wedge}\mathbf{T}),{}^{\wedge\wedge}\mathbf{D})),{}^{\wedge}\mathbf{s})),\mathbf{n}),$
$\mathbf{A}^{0}_{et}(\mathbf{A}^{0}_{(st)(et)}(\mathbf{H},\mathbf{A}^{1}_{et}(\mathbf{A}^{1}_{(st)(et)}({}^{\wedge}\mathbf{S},\mathbf{A}^{2}_{((et)t}(\mathbf{A}^{2}_{(et)((et)t)}({}^{\wedge\wedge}\mathbf{ALL},{}^{\wedge}{}^{2}_{1}{}^{\vee}{}^{2}\,{}^{\wedge\wedge}\mathbf{T}),{}^{\wedge\wedge}\mathbf{D})),{}^{\wedge}\mathbf{s})),\mathbf{n}),$
$\mathbf{A}^{0}_{et}(\mathbf{A}^{0}_{(st)(et)}(\mathbf{H},\mathbf{A}^{1}_{et}(\mathbf{A}^{1}_{(st)(et)}({}^{\wedge}\mathbf{S},\mathbf{A}^{2}_{((et)t}(\mathbf{A}^{2}_{(et)((et)t)}({}^{\wedge\wedge}\mathbf{ALL},{}^{\wedge}{}^{2}_{2}{}^{\vee}{}^{2}\,{}^{\wedge\wedge}\mathbf{T}),{}^{\wedge\wedge}\mathbf{D})),{}^{\wedge}\mathbf{s})),\mathbf{n})\}$

• $\Big[\!\!\Big[\mathbf{A}^{0}_{et}(\mathbf{A}^{0}_{(st)(et)}(\mathbf{H},\mathbf{A}^{1}_{et}(\mathbf{A}^{1}_{(st)(et)}({}^{\wedge}\mathbf{S},\mathbf{A}^{2}_{((et)t}(\mathbf{A}^{2}_{(et)((et)t)}({}^{\wedge\wedge}\mathbf{ALL},{}^{\wedge}{}^{2}_{0}{}^{\vee}{}^{2}\,{}^{\wedge\wedge}\mathbf{T}),{}^{\wedge\wedge}\mathbf{D})),{}^{\wedge}\mathbf{s})),\mathbf{n})\Big]\!\!\Big]^{\mathbb{M},i,g}$

$\stackrel{A8}{=}$ $\Big[\!\!\Big[\mathbf{A}^{0}_{(st)(et)}(\mathbf{H},\mathbf{A}^{1}_{et}(\mathbf{A}^{1}_{(st)(et)}({}^{\wedge}\mathbf{S},\mathbf{A}^{2}_{((et)t}(\mathbf{A}^{2}_{(et)((et)t)}({}^{\wedge\wedge}\mathbf{ALL},{}^{\wedge}{}^{2}_{0}{}^{\vee}{}^{2}\,{}^{\wedge\wedge}\mathbf{T}),{}^{\wedge\wedge}\mathbf{D})),{}^{\wedge}\mathbf{s}))\Big]\!\!\Big]^{\mathbb{M},i,g}([\![\mathbf{n}]\!]^{\mathbb{M},i,g})$

$\stackrel{A8}{=}$ $[\![\mathbf{H}]\!]^{\mathbb{M},i,g}([\![\mathbf{n}]\!]^{\mathbb{M},i,g}, \Big[\!\!\Big[\mathbf{A}^{1}_{et}(\mathbf{A}^{1}_{(st)(et)}({}^{\wedge}\mathbf{S},\mathbf{A}^{2}_{((et)t}(\mathbf{A}^{2}_{(et)((et)t)}({}^{\wedge\wedge}\mathbf{ALL},{}^{\wedge}{}^{2}_{0}{}^{\vee}{}^{2}\,{}^{\wedge\wedge}\mathbf{T}),{}^{\wedge\wedge}\mathbf{D})),{}^{\wedge}\mathbf{s})\Big]\!\!\Big]^{\mathbb{M},i,g})$

$\stackrel{\eta\ conv.}{=}$ $[\![\mathbf{H}]\!]^{\mathbb{M},i,g}([\![\mathbf{n}]\!]^{\mathbb{M},i,g}, \lambda j.\ \Big[\!\!\Big[\mathbf{A}^{1}_{et}(\mathbf{A}^{1}_{(st)(et)}({}^{\wedge}\mathbf{S},\mathbf{A}^{2}_{((et)t}(\mathbf{A}^{2}_{(et)((et)t)}({}^{\wedge\wedge}\mathbf{ALL},{}^{\wedge}{}^{2}_{0}{}^{\vee}{}^{2}\,{}^{\wedge\wedge}\mathbf{T}),{}^{\wedge\wedge}\mathbf{D})),{}^{\wedge}\mathbf{s})\Big]\!\!\Big]^{\mathbb{M},i,g}(j))$

$\stackrel{A8}{=}$ $[\![\mathbf{H}]\!]^{\mathbb{M},i,g}([\![\mathbf{n}]\!]^{\mathbb{M},i,g}, \lambda j. \Big[\!\!\Big[\mathbf{A}^{1}_{(st)(et)}({}^{\wedge}\mathbf{S},\mathbf{A}^{2}_{((et)t}(\mathbf{A}^{2}_{(et)((et)t)}({}^{\wedge\wedge}\mathbf{ALL},{}^{\wedge}{}^{2}_{0}{}^{\vee}{}^{2}\,{}^{\wedge\wedge}\mathbf{T}),{}^{\wedge\wedge}\mathbf{D}))\Big]\!\!\Big]^{\mathbb{M},i,g}(j)\ ([\![{}^{\wedge}\mathbf{s}]\!]^{\mathbb{M},i,g}(j)))$

$\stackrel{IL}{=}$ $[\![\mathbf{H}]\!]^{\mathbb{M},i,g}([\![\mathbf{n}]\!]^{\mathbb{M},i,g}, \lambda j. \Big[\!\!\Big[\mathbf{A}^{1}_{(st)(et)}({}^{\wedge}\mathbf{S},\mathbf{A}^{2}_{((et)t}(\mathbf{A}^{2}_{(et)((et)t)}({}^{\wedge\wedge}\mathbf{ALL},{}^{\wedge}{}^{2}_{0}{}^{\vee}{}^{2}\,{}^{\wedge\wedge}\mathbf{T}),{}^{\wedge\wedge}\mathbf{D}))\Big]\!\!\Big]^{\mathbb{M},i,g}(j)\ ([\![\mathbf{s}]\!]^{\mathbb{M},j,g}))$

$$\overset{A8}{=}\quad \llbracket \mathbf{H} \rrbracket^{\mathbb{M},i,g}(\llbracket \mathbf{n} \rrbracket^{\mathbb{M},i,g}, \lambda j.\, \llbracket {}^\wedge\mathbf{S} \rrbracket^{\mathbb{M},i,g}(j)(\llbracket \mathbf{A}^{2}_{((et)t)}(\mathbf{A}^{2}_{(et)((et)t)}({}^{\wedge\wedge}\mathbf{ALL}, {}^{\wedge}{}^{2}_{0}{\vee}^{2}{}^{\wedge\wedge}\mathbf{T}), {}^{\wedge\wedge}\mathbf{D}) \rrbracket^{\mathbb{M},i,g}(j))(\llbracket \mathbf{s} \rrbracket^{\mathbb{M},j,g}))$$

$$\overset{IL}{=}\quad \llbracket \mathbf{H} \rrbracket^{\mathbb{M},i,g}(\llbracket \mathbf{n} \rrbracket^{\mathbb{M},i,g}, \lambda j.\, \llbracket \mathbf{S} \rrbracket^{\mathbb{M},j,g}(\llbracket \mathbf{A}^{2}_{((et)t)}(\mathbf{A}^{2}_{(et)((et)t)}({}^{\wedge\wedge}\mathbf{ALL}, {}^{\wedge}{}^{2}_{0}{\vee}^{2}{}^{\wedge\wedge}\mathbf{T}), {}^{\wedge\wedge}\mathbf{D}) \rrbracket^{\mathbb{M},i,g}(j))(\llbracket \mathbf{s} \rrbracket^{\mathbb{M},j,g}))$$

$$\overset{\eta\ \text{conv.}}{=}\quad \llbracket \mathbf{H} \rrbracket^{\mathbb{M},i,g}(\llbracket \mathbf{n} \rrbracket^{\mathbb{M},i,g}, \lambda j.\, \llbracket \mathbf{S} \rrbracket^{\mathbb{M},j,g}(\lambda k.\, \llbracket \mathbf{A}^{2}_{((et)t)}(\mathbf{A}^{2}_{(et)((et)t)}({}^{\wedge\wedge}\mathbf{ALL}, {}^{\wedge}{}^{2}_{0}{\vee}^{2}{}^{\wedge\wedge}\mathbf{T}), {}^{\wedge\wedge}\mathbf{D}) \rrbracket^{\mathbb{M},i,g}(j)(k))(\llbracket \mathbf{s} \rrbracket^{\mathbb{M},j,g}))$$

$$\overset{A8}{=}\quad \llbracket \mathbf{H} \rrbracket^{\mathbb{M},i,g}(\llbracket \mathbf{n} \rrbracket^{\mathbb{M},i,g}, \lambda j.\, \llbracket \mathbf{S} \rrbracket^{\mathbb{M},j,g}(\lambda k.\, \llbracket \mathbf{A}^{2}_{(et)((et)t)}({}^{\wedge\wedge}\mathbf{ALL}, {}^{\wedge}{}^{2}_{0}{\vee}^{2}{}^{\wedge\wedge}\mathbf{T}) \rrbracket^{\mathbb{M},i,g}(j)(k)(\llbracket {}^{\wedge\wedge}\mathbf{D} \rrbracket^{\mathbb{M},i,g})(j)(k))(\llbracket \mathbf{s} \rrbracket^{\mathbb{M},j,g}))$$

$$\overset{A8}{=}\quad \llbracket \mathbf{H} \rrbracket^{\mathbb{M},i,g}(\llbracket \mathbf{n} \rrbracket^{\mathbb{M},i,g}, \lambda j.\, \llbracket \mathbf{S} \rrbracket^{\mathbb{M},j,g}(\lambda k.\, \llbracket {}^{\wedge\wedge}\mathbf{ALL} \rrbracket^{\mathbb{M},i,g}(j)(k)(\llbracket {}^{\wedge}{}^{2}_{0}{\vee}^{2}{}^{\wedge\wedge}\mathbf{T} \rrbracket^{\mathbb{M},i,g}(j)(k))(\llbracket {}^{\wedge\wedge}\mathbf{D} \rrbracket^{\mathbb{M},j,g})(j)(k))(\llbracket \mathbf{s} \rrbracket^{\mathbb{M},j,g}))$$

$$\overset{A1}{=}\quad \llbracket \mathbf{H} \rrbracket^{\mathbb{M},i,g}(\llbracket \mathbf{n} \rrbracket^{\mathbb{M},i,g}, \lambda j.\, \llbracket \mathbf{S} \rrbracket^{\mathbb{M},j,g}(\lambda k.\, \llbracket \mathbf{ALL} \rrbracket^{\mathbb{M},k,g}(\llbracket {}^{\wedge}{}^{2}_{0}\mathbf{T} \rrbracket^{\mathbb{M},i,g}(j)(k))(\llbracket \mathbf{D} \rrbracket^{\mathbb{M},k,g}))(\llbracket \mathbf{s} \rrbracket^{\mathbb{M},j,g}))$$

$$\overset{A6}{=}\quad \llbracket \mathbf{H} \rrbracket^{\mathbb{M},i,g}(\llbracket \mathbf{n} \rrbracket^{\mathbb{M},i,g}, \lambda j.\, \llbracket \mathbf{S} \rrbracket^{\mathbb{M},j,g}(\lambda k.\, \llbracket \mathbf{ALL} \rrbracket^{\mathbb{M},k,g}(\llbracket \mathbf{T} \rrbracket^{\mathbb{M},i,g})(\llbracket \mathbf{D} \rrbracket^{\mathbb{M},k,g}))(\llbracket \mathbf{s} \rrbracket^{\mathbb{M},j,g}))$$

$$\overset{not.}{=}\quad H_i(\mathrm{n}, \lambda j.\, S_j(\lambda k.\, T_i \subseteq D_k))(\mathrm{s}))$$

- $$\llbracket \mathbf{A}^{0}_{et}(\mathbf{A}^{0}_{(st)(et)}(\mathbf{H},\mathbf{A}^{1}_{et}(\mathbf{A}^{1}_{(st)(et)}({}^\wedge\mathbf{S},\mathbf{A}^{2}_{((et)t)}(\mathbf{A}^{2}_{(et)((et)t)}({}^{\wedge\wedge}\mathbf{ALL},{}^{\wedge}{}^{2}_{1}{\vee}^{2}{}^{\wedge\wedge}\mathbf{T}),{}^{\wedge\wedge}\mathbf{D})),{}^\wedge\mathbf{s})),\mathbf{n}) \rrbracket^{\mathbb{M},i,g}$$

$$= \quad \ldots$$

$$= \quad H_i(\mathrm{n}, \lambda j.\, S_j(\lambda k.\, T_j \subseteq D_k))(\mathrm{s}))$$

- $$\llbracket \mathbf{A}^{0}_{et}(\mathbf{A}^{0}_{(st)(et)}(\mathbf{H},\mathbf{A}^{1}_{et}(\mathbf{A}^{1}_{(st)(et)}({}^\wedge\mathbf{S},\mathbf{A}^{2}_{((et)t)}(\mathbf{A}^{2}_{(et)((et)t)}({}^{\wedge\wedge}\mathbf{ALL},{}^{\wedge}{}^{2}_{2}{\vee}^{2}{}^{\wedge\wedge}\mathbf{T}),{}^{\wedge\wedge}\mathbf{D})),{}^\wedge\mathbf{s})),\mathbf{n}) \rrbracket^{\mathbb{M},i,g}$$

$$= \quad \ldots$$

$$= \quad H_i(\mathrm{n}, \lambda j.\, S_j(\lambda k.\, T_k \subseteq D_k))(\mathrm{s})) \qquad = \llbracket\, |\,(36)\,|\, \rrbracket^{\mathbb{M},i_0,g}$$