Interpreting Semantic Theories

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1. *Semantic values*

(a) Two theories may assign different semantic values to an expression and still agree on its meaning.
(b) They may disagree on the meaning of an expression to which they assign the same semantic value.

(i) What the meanings of linguistic expressions (if not their semantic values)?
(ii) What is synonymy (if not coincidence of semantic values)?
(iii) What does it mean for two semantic theories to be *equivalent* (given that they both assign semantic values)?

2. *Definitions*

Syntactic terms
either lexical *or* of the form $\Gamma(\Delta_1, \ldots, \Delta_n)$
where Γ is a syntactic *constructor* (indicating a syntactic construction) and the Δ_i are syntactic terms.

Compositional interpretation
$| \Gamma(\Delta_1, \ldots, \Delta_n) | = |\Gamma| (|\Delta_1|, \ldots, |\Delta_n|)$

[pragmatics] interface
$dom([\![\cdot]\!]) \subseteq rge(|\cdot|)$

An *interpreted semantic theory* is a triple $\mathscr{T} = (\mathscr{A}, |\cdot|, [\![\cdot]\!])$, where $\mathscr{A}$ is a [syntactic] term algebra, $|\cdot|$ is a homomorphic [semantic] value assignment on $\mathscr{A}$ and $[\![\cdot]\!]$ is a [pragmatic] interpretation function whose domain is included in the range of $|\cdot|$.

Let $\mathscr{T}_1 = (\mathscr{A}_1, |\cdot|_1, [\![\cdot]\!]_1)$ and $\mathscr{T}_1 = (\mathscr{A}_2, |\cdot|_2, [\![\cdot]\!]_2)$ be semantic theories. Then $\mathscr{T}_1$ is *equivalent to* $\mathscr{T}_2$ in symbols: $\mathscr{T}_1 \approx \mathscr{T}_2$ – just in case the following conditions are met:

- $\mathscr{A}_1 = \mathscr{A}_2$;
- $|\cdot|_1 \cong |\cdot|_2$;
- $[\![\cdot]\!]_1 \circ |\cdot|_1 = [\![\cdot]\!]_2 \circ |\cdot|_2$.

① Two theories may share a complete semantic value assignment and even agree on the interpretation of all values that are interpretable according to both, and yet not be equivalent.

② Two theories may share a complete semantic value assignment and even agree on which values are interpretable, and yet not be equivalent.

③ It is possible for two theories to have disjoint value assignments and still be equivalent.

3. Examples

ad ③ :

(0) **Angie despises Guido**

$\mathcal{T}_1 = (\mathcal{L}_1, |\cdot|_1, [\![\cdot]\!]_1)$

$|\textbf{Angie}|_1$ = Merkel = a

$|\textbf{Guido}|_1$ = Westerwelle = g

$(a,g) \notin |\textbf{despises}|_1$

$|Verb\ Name|_1 = \{x \mid (|Name|_1, x) \in |Verb|_1\}$

$|(0)|_3 = 0$

$[\![\cdot]\!]_1$: identical map on truth values

$\mathcal{T}_2 = (\mathcal{L}_1, |\cdot|_2, [\![\cdot]\!]_1)$

$|Verb|_2 = |Verb|_2^{-1} = \{(y,x) \mid (x,y) \in |Verb|_1\}$

$|Verb\ Name|_2 = \{x \mid (|Name|_2, x) \in |Verb|_2\}$

$\mathcal{T}_3 = (\mathcal{L}_1, |\cdot|_3, [\![\cdot]\!]_1)$

$|Name|_3 = \{M \subseteq U \mid |Name|_1 \in M\}$

$\mathcal{T}_4 = (\mathcal{L}_1, |\cdot|_4, [\![\cdot]\!]_1)$

$|N|_4 = |N|_3$; $|V|_4 = |V|_2$; $|S|_4 = 1 - |S|_1$

$|P|_4 = \{(x,v) \in U \times \{0,1\} \mid u = 1 \text{ iff } x \in |P|_1\}$

$[\![\cdot]\!]_4 = \{(0,\text{TRUE}), (1,\text{FALSE})\}$

$\Rightarrow \quad \mathcal{T}_1 \approx \mathcal{T}_4$

$\mathcal{T}_n^i = \left(\mathcal{L}_1, |\cdot|_n^i, [\![\cdot]\!]_n^i\right)$,

where i is a point in Logical Space and n=1,…4.

$\mathcal{T}_n^* = \left(\mathcal{L}_1, |\cdot|_n^*, [\![\cdot]\!]_n^*\right)$,

where $|X|_n^*(i) = |X|_n^i$ for any $\mathcal{L}_1$-expression X and point i and

$[\![S]\!]_n^* = |S|_n^*$ iff S is an $\mathcal{L}_1$-sentence S.

$\Rightarrow \quad \mathcal{T}_1^* \approx \mathcal{T}_2^* \approx \mathcal{T}_3^* \approx \mathcal{T}_4^*$

ad ①:

$\Rightarrow \quad \mathscr{T}_1^{+} \not\approx \mathscr{T}_2^{*}$,

where $[\![\cdot]\!]_n^{*} \subseteq [\![\cdot]\!]_n^{+}$ and

$[\![P]\!]_n^{+}$ is (or corresponds to) $|P|_n^{i}$ if P is an $\mathscr{A}_1$-predicate.

(1) **A farmer owns a donkey.**

$R_1 = \hat{x}\hat{y}.\, x$ is a farmer and y is a donkey belonging to x.

(2) **He keeps it in a stable.**

(3) **It is not the case that no farmer owns a donkey.**

$\mathscr{T}_1^{d} = \left(\mathscr{A}_1^{d}, |\cdot|_1^{d}, [\![\cdot]\!]_1^{d}\right)$

where $|(1)_S|_1^{d} = |(1)_D|_1^{d} = R_1$, and $[\![R_1]\!]_1^{d} = R_1$

$\mathscr{T}_2^{d} = \left(\mathscr{A}_1^{d}, |\cdot|_2^{d}, [\![\cdot]\!]_2^{d}\right)$

where $|(1)_D|_2^{d} = R_1$, $[\![R_1]\!]_2^{d} = R_1$, but $|(1)_S|_2^{d} \notin dom\left([\![\cdot]\!]_2^{d}\right)$

$\Rightarrow \quad \mathscr{T}_1^{d} \not\approx \mathscr{T}_2^{d}$

ad ②:

$\mathscr{T}_3^{d} = \left(\mathscr{A}_1^{d}, |\cdot|_2^{d}, [\![\cdot]\!]_3^{d}\right)$

where $[\![R_1]\!]_3^{d} = \exists(R_1) = [\![\,|(3)_D|_2^{d}\,]\!]_3^{d}$ ∃XISTENTIAL CLOSURE

$\Rightarrow \quad \mathscr{T}_2^{d} \not\approx \mathscr{T}_3^{d}$

(4) **The antique dealer convinced the farmer that <u>only very few of the highly valuable pieces were worth anything</u>.**

4. *Whither Synonymy?*

(*I*) $[\![|\Delta|]\!] = [\![|\Delta'|]\!]$ sharing *I*nterpretation

(*V*) $|\Delta| = |\Delta'|$ sharing *V*alue

(*B*) $|\Delta| = |\Delta'|$ & $[\![|\Delta|]\!] = [\![|\Delta'|]\!]$ sharing *Both* value & interpretation

(*I*+) $[\![|\Delta|_1]\!]_1 = [\![|\Delta'|_2]\!]_2$

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