

Explicit and implicit world variables

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Abstract concrete

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1 Introduction

- (1) John loved Mary.
- (2) $\mathbf{P} L(j,m)$
- (3) $(\exists t' < t) \mathbf{L}_{t'}(j, m)$
- (4) John knows that it is raining.
- (5) $\Box_j r$
- (6) $(\forall w)[wK_j w' \rightarrow r_{w'}]$

2 A Montagovian Perspective

Montague (1970)

2.1 Extensions and Intensions

- (7) $\llbracket \textit{snores} \rrbracket$
= that function S such that, for any extension x of a referential subject N :
 $S(x) = \vdash N \textit{snores} \dashv$
- (8) $\llbracket \textit{nobody} \rrbracket$
= that function Q such that, for any extension E of a predicate P :
 $Q(E) = \vdash \textit{Nobody} P \dashv$
- (9) $\llbracket X \rrbracket$
= that function f such that, for any extension x of a possible sister node Y :
 $f(x) = \llbracket X Y \rrbracket$

* ... wishes you a HAPPY BIRTHDAY, KJELL JOHAN!

- (10) Preconditions for (9)
- a. To begin with, the extension of the sister node Y in (9) must be determined beforehand – either (i) by previous applications of (9), or (ii) by independent assumptions.
 - b. Similarly, (9) can only be applied after the extension of the mother node XY has been determined.
 - $\llbracket XY \rrbracket = \llbracket X \rrbracket(\llbracket Y \rrbracket)$
 - c. The extension of the mother node XY must depend on the extension of Y in the first place: if Y is replaced by some co-extensional Z , then the resulting expression XZ needs to have the same extension as XY .

- (11) $\llbracket \text{believe} \rrbracket$
 = that function B such that, for any sense p (of a sentence S):

$$B(p) = \llbracket \text{believe that } S \rrbracket$$

(12) $f(x) = \llbracket X Y \rrbracket^{w^*} = \llbracket X \rrbracket^{w^*}(\llbracket Y \rrbracket^{w^*})$

- (13) The intension $\llbracket X \rrbracket^\wedge$ of an expression X is that function f such that, for any possible world w :
- $$f(w) = \llbracket X \rrbracket^w$$

- *Sense determines reference.*
- Given the circumstances, sense determines reference.

- (14) John knows that an equilateral triangle has three equal sides.
 (15) John knows that an equiangular triangle has three equal sides.

2.2 Taking Stock: Fregean Principles of Interpretation

- i. Grammar assigns to every expression X its intension (relative to a context of use, to be suppressed), which is a function $\llbracket X \rrbracket^\wedge$ that assigns to any possible world w the extension $\llbracket X \rrbracket^w$ of X relative to w .
- ii. Declarative sentences and referential expressions have basic extensions; all other extensions are (functional) contributions to basic expressions [↪Frege (1884)]
- iii. The (basic) extension of a declarative sentence is a truth value, i.e., a member of $D_t [= \{0, 1\}]$. [↪Frege (1892)]
- iv. The (basic) extension of a referential expression is an individual, i.e., a member of D_e . [↪Frege (1892)]

- v. The functional extension of an expression X is its contribution to the extension of certain ‘canonical’ expressions in which X occurs in an extensional position – i.e. a function from its sister extensions ($[\vec{Y}]^w, [\vec{Z}]^w$) to the corresponding mother extensions $[\vec{Y} X \vec{Z}]^w$. [↔Frege (1891)]
- vi. The intensions of primitive expressions are listed in the lexicon. [↔Carnap (1947)]
- vii. The extension of a complex expression $X = \mathcal{F}(Y_1, \dots, Y_n)$ results from combining the (a) extensions or (b) intensions of its immediate parts Y_i , depending on whether i is an (a) extensional position or (b) not, i.e., whether $\mathcal{F}(Y_1, \dots, Y_i, \dots, Y_n)$ and $X = \mathcal{F}(Y_1, \dots, Z, \dots, Y_n)$ are co-extensional whenever Y_i and Z are. [↔Frege (1892)]
- viii. As a consequence, all possible extensions can be arranged in a hierarchy $(D_a)_{a \in IT}$, where IT is the smallest set S containing t and e as well as (ab) and (sb) , whenever $a, b \in S$; $D_{(ab)} = D_b^{D_a}$ contains all functions with domain D_a and range included in D_b ; and D_s (which is not part of the hierarchy) is Logical Space. [↔(Montague 1970: sec. 4)]

2.3 Indirect interpretation

- (16) *Compositional translation: basic strategy*
 - a. Any natural language expression X is assigned a Ty2-formula $|X|$ that denotes X ’s extension.
 - b. Lexical expressions $|X|$ translate as $c(i)$, where c denotes the intension of X and i is a fixed world-variable.
 - c. Any n -place syntactic construction corresponds to a Ty2-formula with n free variables.
 - d. Complex expressions $X_1 \dots X_n$ are translated by Ty2-formulae in which (free) variables $x_1 \dots x_n$ are replaced by the translations $|X_1| \dots |X_n|$ of the constituents.
- (17) *Consequences of 2.2 for Translations $|X|$*
 - a. $|X|$ contains only constants of types (sa) where $a \in IT$.
 - b. $|X|$ contains at most one free variable of type s .
 - c. $|X|$ contains only parameters of types $a \in IT \cup \{s\}$.
- (18) Theorem [↔Zimmermann (1989)]
Any $|X|$ satisfying (18-a.)–(18-c.) can be expressed without (explicit) variables of type s .

3 Bäuerle's Problem

Bäuerle (1983)

- (19) Syd sees that every band member is drinking.
- (20) a. $S_i(s, \lambda j. \forall (B_j, D_j))$ in situ
 b. $\forall (B_i, S_i(.S_i(s, \lambda j. D_j(x))))$ wide scope
 c. $S_i(s, \lambda j. \forall (B_i, D_j))$ Bäuerle
- (21) a. Norman hears that Syd sees that every band member is drinking.
 b. $H^0(n, \wedge S^1(s, \wedge (\forall x)[B(x) \rightarrow D(x)]))$ Frege:1892; cf. Parsons (1981)
 $\equiv H_i^0(n, \lambda j. S_j^1(s, \lambda j. \lambda k. [\lambda j. \lambda k. \forall (B_k)](j)(k)[\lambda j. \lambda k. \forall (B_k)](j)(k)))$

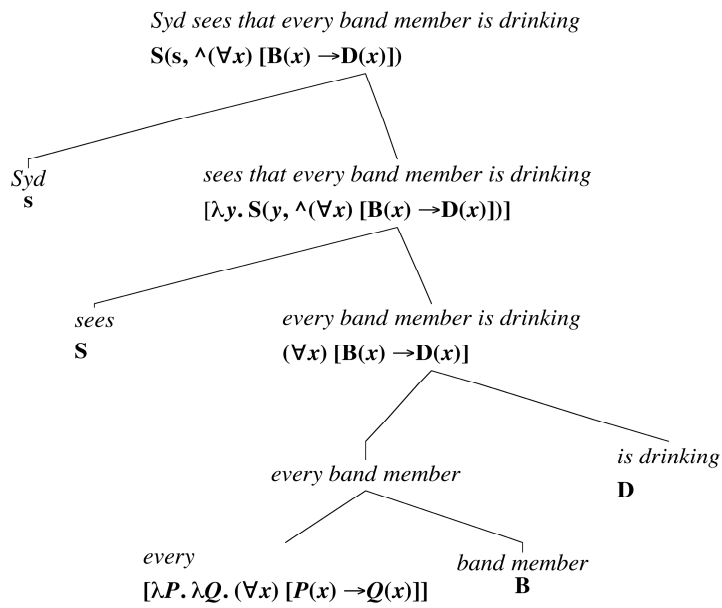
- From rigid intensions ...
 $\llbracket \wedge^n \alpha \rrbracket^{w_0}(w_1), \dots (w_n) = \llbracket \alpha \rrbracket^{w_n}$
- ... to twisted senses
 $\llbracket \wedge_m^n \alpha \rrbracket^{w_0}(w_1), \dots (w_n) = \llbracket \alpha \rrbracket^{w_m}$

References

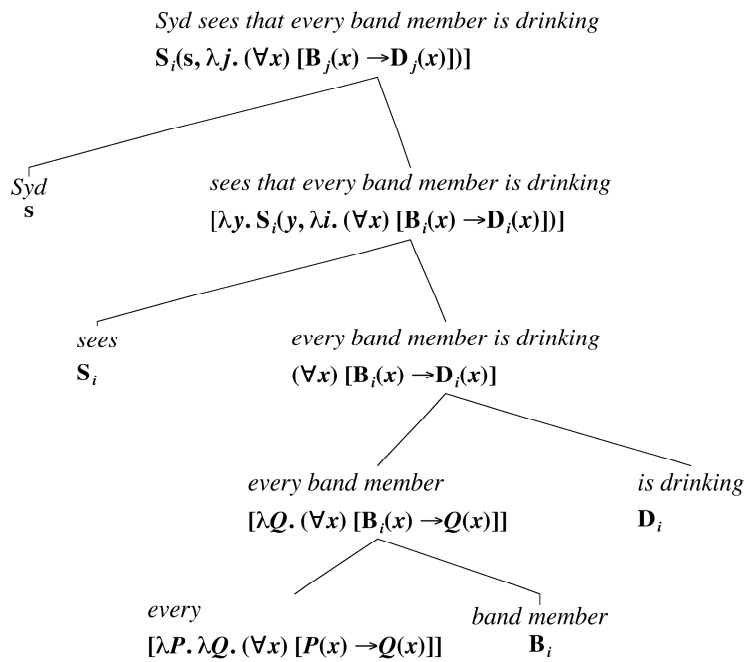
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Appendix: Trees

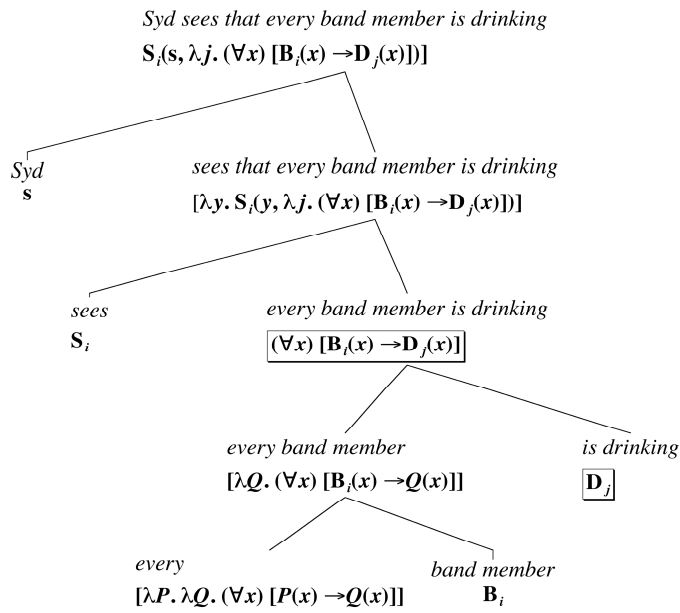
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Groenendijk & Stokhof (1982)

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