

Models and Worlds in Linguistic Semantics

Thomas Ede Zimmermann
Frankfurt

1 Introduction

(1) What this talk is about:

👍 Model-theoretic possible worlds semantics of natural language

↪ Montague (1970)

👍 Alternative Semantics of focus

↪ Rooth (1985)

(2) What this talk is NOT about:

👎 extensional models

👎 illustrative uses of models

👎 loose uses of ‘models’

👎 model-theoretic meta-semantics

👎 direct vs. indirect interpretation

👎 context dependence

👎 raw data of semantics

Background:

- disambiguation, LF, compositionality
- types
- terminology: models vs. (‘Fregean’) interpretations

2 Model-theoretic ‘Semantics’

2.1 Relative Truth

My thesis is only that there are important differences between theories of relative, and of absolute, truth, and the differences make theories of the two sorts appropriate as answers to different questions.

(Davidson 1973: 79)







2.2 Models vs. ‘Small Worlds’

- (3) $M = (D_M, W_M, F_M, \dots)$ – where D_M and W_M are *arbitrary* non-empty sets
- $\llbracket S \rrbracket^{M,w} \in \{0, 1\}$
 - $\llbracket S \rrbracket^M = \{w \in W_M \mid \llbracket S \rrbracket^{M,w} = 1\}$
 - $\llbracket S \rrbracket = \lambda M. \{w \in W_M \mid \llbracket S \rrbracket^{M,w} = 1\}$

(4) Semantic (?) Levels

- a. $R(S_1, S_2)$ holds *locally* (in M at w) iff $R_l(\llbracket S_1 \rrbracket^{M,w}, \llbracket S_2 \rrbracket^{M,w})$
 - S_1 locally [materially] implies S_2 (in M at w) iff $\llbracket S_1 \rrbracket^{M,w} \leq \llbracket S_2 \rrbracket^{M,w}$;
 - S_1 is locally distinct from S_2 (in M at w) iff $\llbracket S_1 \rrbracket^{M,w} \neq \llbracket S_2 \rrbracket^{M,w}$.
- b. $R(S_1, S_2)$ holds *regionally* (in M) iff $R_r(\llbracket S_1 \rrbracket_M, \llbracket S_2 \rrbracket_M)$.
 - S_1 regionally [strictly] implies S_2 (in M) iff $\llbracket S_1 \rrbracket^M \subseteq \llbracket S_2 \rrbracket^M$;
 - S_1 is regionally distinct from S_2 (in M) iff $\llbracket S_1 \rrbracket^M \neq \llbracket S_2 \rrbracket^M$.
- c. $R(S_1, S_2)$ holds *globally* iff $R_g(\llbracket S_1 \rrbracket^M, \llbracket S_2 \rrbracket^M)$
 - S_1 globally implies [logically entails] S_2 iff $\llbracket S_1 \rrbracket \sqsubseteq \llbracket S_2 \rrbracket$;
 - S_1 is globally distinct from S_2 iff $\llbracket S_1 \rrbracket \neq \llbracket S_2 \rrbracket$.

(5) Model Space vs. Logical Space(s)

-  Model Space is closed under (arbitrary) isomorphisms, ...
-  Logical Spaces are not – not even necessarily under automorphisms.
-  Cross-model identities are (mostly) meaningless, ... need for interpretation
-  cross-world identity is identity. \approx haecceitism
-  Model Space ought to be small, ... intended models
-  Logical Spaces need to be large. see below

(6) Only John likes Mary.

\therefore Bill doesn't like Mary.

(7) Rigidity


- a. $(\exists M)(\exists w \in W_M) \llbracket John \rrbracket^{M,w} = \llbracket Bill \rrbracket^{M,w}$ Oops!
- b. $(\forall M, M')(\forall w, w' \in W_M) \llbracket John \rrbracket^{M,w} = \llbracket John \rrbracket^{M',w'} (= \text{John})$ ζ (6-a)
- c. $(\forall M)(\forall w, w' \in W_M) \llbracket John \rrbracket^{M,w} = \llbracket John \rrbracket^{M,w'} (= \text{'John}_M)$ cf. Fig. 1

(3) for all $a \in \text{BST}$, the expressions

$$\begin{aligned}
 j(\text{entity}) &\equiv \lambda u [u \equiv u], \\
 j(\text{be}) &\equiv \lambda Q \lambda \mathcal{P} \mathcal{D} \{ \hat{u} Q \{ \hat{v} [u \equiv v] \} \}, \\
 \forall u (j(a) &\equiv \hat{P} P \{ u \})
 \end{aligned}$$

are true sentences of L_0 with respect to $\langle \mathcal{B}', \langle i_0, j_0 \rangle \rangle$, where u, v, P, \mathcal{D}, Q are $\nu_0, e, \nu_1, e, \nu_0, \langle e, \langle e, t \rangle \rangle, \nu_0, s(4), \nu_1, s(4)$ respectively.

Figure 1 (Montague 1970: 393)

 Rigidity is stability across worlds, not models.

3 Alternative Semantics: a Case Study

Rooth (1985)

3.1 Quantification over propositions

- (8) a. Only Mary is asleep.
 b. $(\forall p)[[\forall p \wedge (\exists x) p = \wedge \mathbf{S}(x)] \rightarrow p = \wedge \mathbf{S}(\mathbf{m})]$
 c. $(\forall x)[\mathbf{S}(x) \rightarrow x = \mathbf{m}]$
- (9) $(\forall x)(\forall y)[x \neq y \rightarrow \wedge \mathbf{S}(x) \neq \wedge \mathbf{S}(y)]$ Postulate
- (10) a. John only met MARY.
 b. $(\forall p)[[\forall p \wedge (\exists x) p = \wedge \mathbf{M}(\mathbf{j}, x)] \rightarrow p = \wedge \mathbf{M}(\mathbf{j}, \mathbf{m})]$
 c. $(\forall x_1)(\forall x_2)(\forall y_1)(\forall y_2)[(x_1, x_2) \neq (y_1, y_2) \rightarrow \wedge \mathbf{M}(x_1, x_2) \neq \wedge \mathbf{M}(y_1, y_2)]$
- (11) a. John only introduces MARY to Sue.
 b. $(\forall p)[[\forall p \wedge (\exists x) p = \wedge \mathbf{I}(\mathbf{j}, x, \mathbf{s})] \wedge \rightarrow p = \wedge \mathbf{I}(\mathbf{j}, \mathbf{m}, \mathbf{s})]$
 c. $(\forall x_1)(\forall x_2)(\forall x_3)(\forall y_1)(\forall y_2)(\forall y_3)[(x_1, x_2, x_3) \neq (y_1, y_2, y_3) \rightarrow \wedge \mathbf{I}(x_1, x_2, x_3) \neq \wedge \mathbf{I}(y_1, y_2, y_3)]$
- (12) a. Only Mary is both drunk and asleep.
 b. $(\forall p)[[\forall p \wedge (\exists x) p = \wedge [\mathbf{D}(x) \wedge \mathbf{S}(x)]] \rightarrow p = \wedge [\mathbf{D}(\mathbf{m}) \wedge \mathbf{S}(\mathbf{m})]]$
 c. $(\forall x)(\forall y)[x \neq y \rightarrow \wedge [\mathbf{D}(x) \wedge \mathbf{S}(x)] \neq \wedge [\mathbf{D}(y) \wedge \mathbf{S}(y)]]$
- (13) a. John only knows that MARY knows that Harry introduces Bill to Sue.
 b. $(\forall p)[[\forall p \wedge (\exists x) p = \wedge \mathbf{K}(\mathbf{j}, \wedge \mathbf{K}(x, \wedge \mathbf{I}(\mathbf{h}, \mathbf{b}, \mathbf{s})))] \rightarrow p = \wedge \mathbf{K}(\mathbf{j}, \wedge \mathbf{K}(\mathbf{m}, \wedge \mathbf{I}(\mathbf{h}, \mathbf{b}, \mathbf{s})))]$
 c. $(\forall x_1) \dots (\forall x_5)(\forall y_1) \dots (\forall y_5)[(x_1, \dots, x_5) \neq (y_1, \dots, y_5) \rightarrow \wedge \mathbf{K}(x_1, \wedge \mathbf{K}(x_2, \wedge \mathbf{I}(x_3, x_4, x_5))) \neq \wedge \mathbf{K}(y_1, \wedge \mathbf{K}(y_2, \wedge \mathbf{I}(y_3, y_4, y_5)))]$
- (14) $\lambda x. [\mathbf{D}(x) \wedge \mathbf{S}(x)]$
- (15) $\lambda x_5. \lambda x_4. \lambda x_3. \lambda x_2. \lambda x_1. \mathbf{K}(x_1, \wedge \mathbf{K}(x_2, \wedge \mathbf{I}(x_3, x_4, x_5)))$
- (16) $(\forall \vec{x})(\forall \vec{y})[\vec{x} \neq \vec{y} \rightarrow [\wedge R\{\vec{x}\}] \neq [\wedge R\{\vec{y}\}]]$

3.2 Extensional Variation by Meaning Postulates

- (17) $(\forall X) \diamond [\mathbf{S} = X]$
- (18) $(\forall R)[[\neg(\exists x)R(x, x)] \rightarrow \diamond [\mathbf{M} = R]]$
- (19) $(\forall S)[[\neg(\exists x)(\exists y)[S(x, y, x) \vee S(x, y, y)]] \rightarrow \diamond [\mathbf{I} = S]]$
- (20) $(\forall X) \diamond [\lambda x. [\mathbf{D}(x) \wedge \mathbf{S}(x)] = X]$
- (21) $(\forall T) \diamond [[\lambda x_5. \lambda x_4. \lambda x_3. \lambda x_2. \lambda x_1. \mathbf{K}(x_1, \wedge \mathbf{K}(x_2, \wedge \mathbf{I}(x_3, x_4, x_5)))] = T]$

3.3 Extensional variation by reflection principles

- (22) a. For any $X \subseteq D$, there is a \mathcal{K}_0 -model $M = (D, W, F_M)$ and a world $w \in W$ such that:

$$F_M(\mathbf{S})(w) = X.$$
b. For any $R \subseteq D^2$ there is a \mathcal{K}_0 -model $M = (D, W, F_M)$ and a world $w \in W$ such that:

$$F_M(\mathbf{M})(w) = R.$$
c. For any $S \subseteq D^3$ there is a \mathcal{K}_0 -model $M = (D, W, F_M)$ and a world $w \in W$ such that:

$$F_M(\mathbf{I})(w) = S.$$

- (23) a. $\neg(\exists x)\mathbf{M}(x, x)$
b. $\neg(\exists x)(\exists y)[\mathbf{I}(x, y, x) \vee \mathbf{I}(x, y, y)]$

- (24) For any irreflexive $R \subseteq D^2$ there is a \mathcal{K}_1 -model $M = (D, W, F_M)$ and a world $w \in W$ such that:

$$F_M(\mathbf{M})(w) = R.$$

- (25) For any irreflexive₃ $S \subseteq D^2$ there is a \mathcal{K}_1 -model $\mathfrak{M} = (D, W, F_{\mathfrak{M}})$ and a world $w \in W$ such that:

$$F_{\mathfrak{M}}(\mathbf{I})(w) = R.$$

(26) $\{\varphi \mid M \models_w \varphi\} = \{\varphi \mid M^* \models_{w^*} \varphi\}$

(27) Mary is asleep.

- (28) a. $M_0 \not\models_w \mathbf{S}(\mathbf{m})$, for any M_0 -world w ;
b. $M_1 \models_{w'} \mathbf{S}(\mathbf{m})$, for some M_1 -world w' .

- (29) a. $M_0 \models_w \neg \diamond \mathbf{S}(\mathbf{m})$
b. $M_1 \models_{w'} \diamond \mathbf{S}(\mathbf{m})$

- (30) a. $M^* \models_{w_0} \neg \diamond \mathbf{S}(\mathbf{m})$
b. $M^* \not\models_{w_1} \diamond \mathbf{S}(\mathbf{m})$

(31) $M^* \models_{w^*} [[\neg \diamond \mathbf{S}(\mathbf{m})] \wedge \diamond \mathbf{S}(\mathbf{m})]$

3.4 Limits of propositional quantification

(32) $(\forall \vec{x})(\forall \vec{y})[\vec{x} \neq \vec{y} \rightarrow [{}^\wedge R\{\vec{x}\}] \neq [{}^\wedge R\{\vec{y}\}]]$ [= (16)]

(33) $(\forall R)(\forall \vec{x})(\forall \vec{y})[\vec{x} \neq \vec{y} \rightarrow [{}^\wedge R\{\vec{x}\}] \neq [{}^\wedge R\{\vec{y}\}]]$

(34) $(\forall R)[\wp(R) \rightarrow (\forall \vec{x})(\forall \vec{y})[\vec{x} \neq \vec{y} \rightarrow {}^\wedge R\{\vec{x}\} \neq {}^\wedge R\{\vec{y}\}]]$

(35) Only three is an odd number .

- (36) $(\forall x)[\mathbf{O}(x) \rightarrow x = \mathbf{3}]$
- (37) $(\forall p)[[\bigvee p \wedge (\exists x)p = \wedge \mathbf{O}(x)] \rightarrow p = \wedge \mathbf{O}(\mathbf{3})]$
- (38) Only Mary is one of John and Mary and exactly as tall as either one.
- (39) $(\forall x)[[x = \mathbf{j} \vee x = \mathbf{m}] \wedge (\forall y)[[y = \mathbf{j} \vee y = \mathbf{m}] \rightarrow \mathbf{h}(x) = \mathbf{h}(y)] \rightarrow x = \mathbf{m}]$
 $\Rightarrow \mathbf{j} = \mathbf{m}$
- (40) $(\forall p)[[\bigvee p \wedge (\exists x)p = \wedge [[x = \mathbf{j} \vee x = \mathbf{m}] \wedge (\forall y)[[y = \mathbf{j} \vee y = \mathbf{m}] \rightarrow \mathbf{h}(x) = \mathbf{h}(y)]]] \rightarrow p = \wedge [[\mathbf{m} = \mathbf{j} \vee \mathbf{m} = \mathbf{m}] \wedge (\forall y)[[y = \mathbf{j} \vee y = \mathbf{m}] \rightarrow \mathbf{h}(\mathbf{m}) = \mathbf{h}(y)]]]$
- (41) $(\forall p)[[\bigvee p \wedge p = \wedge [\mathbf{h}(\mathbf{j}) = \mathbf{h}(\mathbf{m})]]] \rightarrow p = \wedge [\mathbf{h}(\mathbf{j}) = \mathbf{h}(\mathbf{m})]]$
- (42) $[(\forall x)[\mathbf{S}(x) \rightarrow x = \mathbf{m}] \wedge \mathbf{S}(\mathbf{m})]$
 $\equiv (\forall x)[\mathbf{S}(x) \leftrightarrow x = \mathbf{m}]$
- (43) $[(\forall p)[[\bigvee p \wedge (\exists x)p = \wedge \mathbf{S}(x)] \rightarrow p = \wedge \mathbf{S}(\mathbf{m})] \wedge \mathbf{S}(\mathbf{m})]$
 $\equiv [(\forall p)[[\bigvee p \wedge (\exists x)p = \wedge \mathbf{S}(x)] \leftrightarrow p = \wedge \mathbf{S}(\mathbf{m})]$
- (44) $(\forall x)[\mathbf{O}(x) \leftrightarrow x = \mathbf{3}]$
- (45) $(\forall p)[[\bigvee p \wedge (\exists x)p = \wedge \mathbf{O}(x)] \leftrightarrow p = \wedge \mathbf{O}(\mathbf{3})]$
- (46) $[(\forall p)[[\bigvee p \wedge p = \wedge [\mathbf{h}(\mathbf{j}) = \mathbf{h}(\mathbf{m})]]] \rightarrow p = \wedge [\mathbf{h}(\mathbf{j}) = \mathbf{h}(\mathbf{m})]] \wedge [[\mathbf{m} = \mathbf{j} \vee \mathbf{m} = \mathbf{m}] \wedge (\forall y)[[y = \mathbf{j} \vee y = \mathbf{m}] \rightarrow \mathbf{h}(\mathbf{m}) = \mathbf{h}(y)]]]$
 $\equiv \mathbf{h}(\mathbf{m}) = \mathbf{h}(\mathbf{j})$

3.5 Quantification over alternative properties

- (47) Bill knows only that MARY is one of John and Mary and exactly as tall as either one.
- (48) Bill knows that MARY is one of John and Mary and exactly as tall as either one.
- (49) Bill knows only that x is one of John and Mary and exactly as tall as either one.
- (50) a. John only met MARY. [= (10-a)]
 b. $(\forall p)[[\bigvee p \wedge (\exists y)p = \wedge \mathbf{M}(\mathbf{j}, y)] \rightarrow p = \wedge \mathbf{M}(\mathbf{j}, \mathbf{m})]$ [\approx (10-b)]
 c. $(\forall P)[[P\{\mathbf{j}\} \wedge (\exists y)P = \hat{x} \mathbf{M}(x, y)] \rightarrow P = \hat{x} \mathbf{M}(x, \mathbf{m})]$
- (51) a. Harry only met SUE.
 b. $(\forall P)[[P\{\mathbf{h}\} \wedge (\exists y)P = \hat{x} \mathbf{M}(x, y)] \rightarrow P = \hat{x} \mathbf{M}(x, \mathbf{s})]$
- (52) a. $\lambda y. \lambda Q. (\forall z)[Q\{z\} \rightarrow z = y]$
 b. $\lambda p. \lambda \mathcal{A}. (\forall q)[[\bigvee q \wedge \mathcal{A}(q)] \rightarrow q = p]$
 c. $\lambda x. \lambda P. \lambda \mathcal{S}. (\forall S)[[S\{x\} \wedge \mathcal{S}(S)] \rightarrow S = P]$

3.6 Prospects of Alternative Semantics

$$(53) \quad \lambda E.\lambda x. (\forall y)[E(y) \rightarrow y = x] \quad [\approx (52-a)]$$

$$(54) \quad O^-(\wedge[P\{x\}], \lambda p. (\exists y) p = \wedge[P\{y\}]) \equiv O_{\text{only}}(x, \vee P)$$

$$(55) \quad \lambda P.\lambda x. (\forall y)\mu(P\{x\}) \leq \mu(P\{y\}) \quad \text{Cf. Wilkinson (1996: 194)}$$

$$(56) \quad O^-(\wedge[P\{x\}], \lambda p. (\exists y) p = \wedge[P\{y\}]) \equiv O_{\text{even}}(x, P)$$

(8-a) Only Mary is asleep

$$(8-b) \quad (\forall p)[[\vee p \wedge (\exists x) p = \wedge \mathbf{S}(x)] \rightarrow p = \wedge \mathbf{S}(\mathbf{m})]$$

$$(57) \quad (\forall \pi)[[\downarrow \pi \wedge (\exists x) \pi = (\mathbf{S}, x)] \rightarrow \pi = (\mathbf{S}, \mathbf{m})]$$

$$(8-c) \quad (\forall x)[\mathbf{S}(x) \rightarrow x = \mathbf{m}]$$

References

- Davidson, Donald. 1973. In Defense of Convention T. In Hugues Leblanc (ed.), *Truth, Syntax and Modality*, 76 – 86. Amsterdam: North Holland.
- Montague, Richard. 1970. Universal Grammar. *Theoria* 36(3). 373–398.
- Rooth, Mats. 1985. *Association with Focus*: University of Massachusetts at Amherst dissertation.
- Wilkinson, Karina. 1996. The Scope of *Even*. *Natural Language Semantics* 4(3). 193–215.
- Zimmermann, Thomas Ede. 1999. Meaning Postulates and the Model-Theoretic Approach to Natural Language Semantics. *Linguistics and Philosophy* 22. 529–561.
- Zimmermann, Thomas Ede. 2011. Model-theoretic Semantics. In Claudia Maienborn, Klaus von Heusinger & Paul Portner (eds.), *Semantics. An International Handbook of Natural Language Meaning*, vol. 1, 762–801. Berlin: de Gruyter.
- Zimmermann, Thomas Ede. To appear. Quantification over Alternative Intensions. *Semantics and Pragmatics*.