1. Type Shifting

Classics

$$x \mapsto \lambda P.P(x)$$

$$\boldsymbol{e} \mapsto \boldsymbol{et} \quad \boldsymbol{t}$$

Montague (1970)

$$f \mapsto \lambda P.\ \lambda Q.\lambda x.f(P(x),Q(x))$$

$$\boldsymbol{t(tt)} \mapsto \boldsymbol{et} \quad \boldsymbol{et} \quad \boldsymbol{e} \quad \boldsymbol{t}$$

Montague (1973)

$$R \mapsto \lambda\Omega.\quad \lambda x.\ (\Omega y)\ R(x,y)$$

$$\boldsymbol{e(et)} \mapsto \boldsymbol{(et)t} \quad \boldsymbol{e} \quad \boldsymbol{t}$$

Montague (1973)

$$M \mapsto \lambda P.\lambda x.\ [M(x) \wedge P(x)]$$

$$\boldsymbol{et} \mapsto \boldsymbol{et} \quad \boldsymbol{e} \quad \boldsymbol{t}$$

Montague (1970)

Type shifts are injective

$$M \;\mapsto\; \lambda P.\ \lambda x.\ [M(x) \wedge P(x)]$$

$$\boldsymbol{et} \;\mapsto\; \boldsymbol{et} \quad \boldsymbol{e} \quad \boldsymbol{t}$$

$$M \;\mapsto\; \lambda P.\ \lambda x.\ [M(x) \vee P(x)]$$

$$\boldsymbol{et} \;\mapsto\; \boldsymbol{et} \quad \boldsymbol{e} \quad \boldsymbol{t}$$

Type shifts preserve semantic substance

$$x \mapsto \lambda P.P(x)$$

$$\boldsymbol{e} \mapsto \boldsymbol{et} \quad \boldsymbol{t}$$

$$x \mapsto \lambda P.\neg P(x)$$

$$\boldsymbol{e} \mapsto \boldsymbol{et} \quad \boldsymbol{t}$$

Type shifts do not add semantic properties

$$R \mapsto \lambda\Omega.\ \lambda\Sigma.\ (\Sigma x)\,(\Omega y)\,R(x,y)$$

$$\boldsymbol{e(et)} \mapsto \boldsymbol{(et)t\ \ (et)t\ \ t}$$

Montague (1970)

$$R \mapsto \lambda\Omega.\ \lambda\Sigma.\ (\Delta y)\,(\Sigma x)\,R(x,y)$$

$$\boldsymbol{e(et)} \mapsto \boldsymbol{(et)t\ \ (et)t\ \ t}$$

Type shifts are (true) embeddings

$$R \quad \mapsto \lambda y.\ \lambda x.\ R(y,x)$$

$$\boldsymbol{e(et)} \quad \mapsto \boldsymbol{e} \quad \boldsymbol{e} \quad \boldsymbol{t}$$

$$R \quad \mapsto \lambda g.\, R(g(0),g(1))$$

$$\boldsymbol{e(et)} \quad \mapsto \boldsymbol{te} \quad \boldsymbol{t}$$

Läuchli (1970)

Type shifts are total …

$$P \;\longmapsto\; (\iota x)\; P(x)$$

$$\boldsymbol{et} \;\longmapsto\; \boldsymbol{e}$$

Partee (1987)

… and thus cannot be reversed

$$x \;\longmapsto\; \lambda y.\, y = x$$

$$\boldsymbol{e} \;\longmapsto\; \boldsymbol{e} \quad \boldsymbol{t}$$

Partee (1987)

Type shifts need not be unique

$$V \mapsto \lambda i.\, \lambda f.\, V(i)(f(i))$$

$$\boldsymbol{s(et)} \mapsto \boldsymbol{s} \quad \boldsymbol{se} \quad \boldsymbol{t}$$

Montague (1973)

$$N \mapsto \lambda i.\, \lambda f.\, (\exists x)\, [N(i)(x)\ \&\ f = \lambda i.x]]$$

$$\boldsymbol{s(et)} \mapsto \boldsymbol{s} \quad \boldsymbol{se} \quad \boldsymbol{t}$$

Montague (1973)

Type shifts are global

$$x \ \mapsto \lambda y.\, y = 2x$$

$$\boldsymbol{e} \ \mapsto \boldsymbol{e} \quad \boldsymbol{t}$$

Type shifts are uniform

$$V \quad |\!\rightarrow \lambda i.\, \lambda f.\, V(i)(f(i))$$

$$\mathbf{s(et)} \quad |\!\rightarrow \mathbf{s} \quad \mathbf{se} \quad \mathbf{t}$$

S1

$$N \quad |\!\rightarrow \lambda i.\, \lambda f.\, (\exists x)\ [N(i)(x)\ \&\ f = \lambda i.x]]$$

$$\mathbf{s(et)} \quad |\!\rightarrow \mathbf{s} \quad \mathbf{se} \quad \mathbf{t}$$

S2

$$P \quad |\!\rightarrow \begin{cases} S1(P), \textit{if } P \textit{ is finite} \\ S2(P), \textit{otherwise} \end{cases}$$

$$\mathbf{s(et)} \quad |\!\rightarrow \mathbf{s((se)t)}$$

Type shifts are uniform across domains

$$x \mapsto \begin{Bmatrix} \lambda y.\, x = y \text{ , } if\ D_e\ is\ finite \\ \lambda y.\, x = y \text{ , } if\ D_e\ is\ infinite \end{Bmatrix}$$

$$\boldsymbol{e} \mapsto \quad \boldsymbol{e} \quad \boldsymbol{t}$$

Type shifts are purely formal

… whatever that means:

permutation invariant, …

definable in (fragments of type logic),…

…

van Benthem (1991)

2. Types in dynamic semantics

Indefinites as variables

A farmer owns a donkey

$$(\exists x)\ (\exists y)\ [F(x) \wedge \mathrm{D}(y) \wedge \mathrm{O}(x,y)]$$

$$[F(x) \wedge \mathrm{D}(y) \wedge \mathrm{O}(x,y)]$$

≈ Sentences as predicates

A farmer owns a donkey

$$[\![\varphi]\!] \subseteq U^n$$

$$\underbrace{U \times \ldots \times U}_{n\text{–}mal}$$

Pardon my German

$$B^A = \{f \subseteq (A \times B) \mid f: A \text{–>} B\}$$

$$n = \{m \in \omega \mid m<n\} = \{0,\ldots,n\}$$

$$\{f \subseteq (n \times U) \mid f: n \text{–>} U\}$$

$$U^n \cong \{f \subseteq (\mathrm{X} \times U) \mid f: \mathrm{X} \text{–>} U\} \Leftrightarrow |\mathrm{X}| = n$$

$$[F(x) \wedge \mathrm{D}(y) \wedge \mathrm{O}(x,y)]$$

$\approx$

$$\lambda x.\, \lambda y.\, [F(x) \wedge \mathrm{D}(y) \wedge \mathrm{O}(x,y)]$$

$\boldsymbol{e(et)}$

Cf. Zimmermann (1993)

$$[(\mathrm{F} \times \mathrm{D}) \cap \mathrm{O}]$$

Dynamic conjunction …

A farmer owns a donkey. He beats it

$$[F(x) \wedge D(y) \wedge O(x,y) \wedge B(x,y)]$$

$\approx$

$$\lambda x.\, \lambda y.\, [F(x) \wedge D(y) \wedge O(x,y) \wedge B(x,y)]$$

$$[\ \underbrace{(F \times D) \cap O}_{\textit{A farmer owns a donkey}}\ \underbrace{\cap}_{\text{[AND]}}\ \underbrace{B}_{\textit{He beats it}}\ ]$$

… as intersection

Dynamic conjunction …

$$[F(x) \wedge D(y) \wedge O(x,y) \wedge B(u,v) \wedge u = x \wedge v = y]$$

$$[[\underbrace{(F \times D) \cap O}_{\textit{A farmer owns a donkey}} \underbrace{\times}_{\text{[AND]}} \underbrace{B}_{\textit{He beats it}}] \cap C]$$

A farmer owns a donkey *He beats it*

[AND]

… as Cartesian production

Dynamic conjunction as type-shifted conjunction

Intersection:

$$f \quad \mapsto \lambda P.\, \lambda Q.\, \lambda x. f(P(x), Q(x))$$

$$\boldsymbol{t(tt)} \quad \mapsto \boldsymbol{et} \quad \boldsymbol{et} \quad \boldsymbol{e} \quad \boldsymbol{t}$$

$$f \quad \mapsto \lambda R.\, \lambda S.\, \lambda \vec{x}. f(R(\vec{x}), S(\vec{x}))$$

$$\boldsymbol{t(tt)} \quad \mapsto \boldsymbol{e}^n\boldsymbol{t} \quad \boldsymbol{e}^n\boldsymbol{t} \quad \boldsymbol{e}^n\boldsymbol{t}$$

Cf. Quine (1960)

Product formation:

$$f \quad \mapsto \lambda R.\, \lambda S.\, \lambda \vec{x}. \lambda \vec{y}.\, f(R(\vec{x}), S(\vec{y}))$$

$$\boldsymbol{t(tt)} \quad \mapsto \boldsymbol{e}^n\boldsymbol{t} \quad \boldsymbol{e}^m\boldsymbol{t} \quad \boldsymbol{e}^n \quad \boldsymbol{e}^m \quad \boldsymbol{t}$$

… vs. Bernays (1957)

Donkeys

Every farmer who owns a donkey beats it

Denotations (simplified):

every: $\lambda P.\ \lambda Q.(\forall x)[P(x) \rightarrow Q(x)]$

farmer who owns a donkey: $(\mathrm{F} \times \mathrm{D}) \cap \mathrm{O}$

beats [it]: B

Donkeys

Every farmer who owns a donkey beats it

Types:
every: ***(et)((et)t)***
farmer who owns a donkey: $\boldsymbol{(e^2t)}$
beats [it]: $\boldsymbol{(e^2t)}$
every farmer who owns a donkey $\boldsymbol{((e^2t)t)}$
every $\boldsymbol{(e^2t)((e^2t)t)}$

Need for type shift

from ***(et)((et)t)*** to $\boldsymbol{(e^2t)((e^2t)t)}$

Need for type shift

from ***(et)((et)t)*** to ***(e^{n}t)((e^{n}t)t)***

QUESTION

ARE THERE ANY?

ANSWER: SURE …

$$D \quad \mapsto \lambda R.\ \lambda S.\ D(\lambda x.\,(\exists \vec{y})\ R(x,\vec{y}))(\lambda x.\,(\exists \vec{y})\ S(x,\vec{y}))$$

$$\boldsymbol{(et)((et)t)} \mapsto \boldsymbol{e^n t \quad e^n t \quad t}$$

Need for type shift

from $\boldsymbol{(et)((et)t)}$ to $\boldsymbol{(e^n t)((e^n t)t)}$

QUESTION

ARE THERE ANY?

ANSWER: SURE …

$$D \quad \mapsto \lambda R.\ \lambda S.\ D(\lambda x.\ (\exists \vec{y})\ R(x,\vec{y}))(\lambda x.\ (\exists \vec{y})\ [R(x,\vec{y}) \wedge S(x,\vec{y})])$$

$$\boldsymbol{(et)((et)t)} \quad \mapsto \boldsymbol{e^n t \quad e^n t \quad t}$$

Asymmetric (weak) shift: no proportion problem, no dimes wasted …

Need for type shift

from ***(et)((et)t)*** to ***(e^{n}t)((e^{n}t)t)***

QUESTION

ARE THERE ANY?

How about?

$$D \quad \mapsto \lambda R.\, \lambda S.\, D(\lambda \vec{x}.R(\vec{x}))(\lambda \vec{x}.S(\vec{x}))$$

$$(et)((et)t) \mapsto \boldsymbol{e^n t \;\; e^n t \;\; t}$$

Lewis (1975), **K**amp (1981), **H**eim (1982)

Need for **type shift**

$$D \quad \longmapsto \lambda R.\ \ \lambda S.\ \ D^*(\lambda \vec{x}.\, R(\vec{x}))(\lambda \vec{x}.\, S(\vec{x}))$$

$$\boldsymbol{(et)((et)t)} \quad \longmapsto \boldsymbol{e}^{n}\boldsymbol{t} \quad \boldsymbol{e}^{n}\boldsymbol{t} \quad \boldsymbol{t}$$

… not a **domain shift**

$$D \quad\quad \mapsto \lambda R.\ \lambda S.\ D(\lambda \vec{x}.\ R(\vec{x}))(\lambda \vec{x}.\ S(\vec{x}))$$

$$\boldsymbol{(et)((et)t)} \mapsto \boldsymbol{e^n t \quad e^n t \quad t}$$

Intended **type shift**

$$D \mapsto \quad \lambda R.\, \lambda S.\, D^*(\lambda \vec{x}.\, R(\vec{x}))(\lambda \vec{x}.\, S(\vec{x}))$$

$$\boldsymbol{(et)et)t)}\ \boldsymbol{e^n t}\ \ \boldsymbol{e^n t}\ \boldsymbol{t}$$

$$\equiv$$

$$D \mapsto \quad D^*$$

$$\boldsymbol{(et)((et)t)}\ \boldsymbol{(e^n t)((e^n t)t)}$$

LHK type shift

$$D \qquad \mapsto \lambda R.\ \lambda S.\ D(\lambda \vec{x}.\ R(\vec{x}))(\lambda \vec{x}.\ S(\vec{x}))$$

$$\boldsymbol{(et)((et)t)} \mapsto \boldsymbol{e^{n}t \quad e^{n}t \quad t}$$

… is not unique:

On a domain of *n* individuals we may have:

$$D_1^*(\forall) = \exists^{=n}$$

or:

$$D_2^*(\forall) = \exists^{\geq n}$$

LKH type shift

$$D \quad \mapsto \lambda R.\ \lambda S.\ D(\lambda\vec{x}.\ R(\vec{x}))(\lambda\vec{x}.\ S(\vec{x}))$$

$$\boldsymbol{(et)((et)t)} \quad \mapsto \boldsymbol{e^n t \quad e^n t \quad t}$$

… can be defined for invariant determiners on infinite domain:

$$D_{(et)((et)t)} \approx \alpha(D) \text{ where } \alpha\text{: } D_{e} \to D_{e^n t} \text{ is an isomorphism}$$

… and extended by gerrymandering:

$$D \quad \mapsto \begin{cases} \lambda R.\quad \lambda S.\ \alpha(D)(R)(S),\ \textit{if D is invariant} \\ \lambda R.\quad \lambda S.\ (\exists P)\ (\exists Q)\ [D(P)(Q) \wedge R = \vec{P} \wedge S = \vec{Q}] \end{cases}$$

$$\boldsymbol{(et)((et)t)} \quad \mapsto \quad \boldsymbol{e^n t \quad e^n t \quad t}$$

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