

## Values and Variables

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### 0. *Bäuerle's problem*

Bäuerle (1983)

(1) Syd sees that every band member is drinking.

(2a)  $S_i(s, \lambda j. \forall(B_j, D_j))$

*in situ*

(b)  $\forall(B_i, \lambda x. S_i(s, \lambda j. D_j(x)))$

*wide scope*

(c)  $S_i(s, \lambda j. \forall(B_i, D_j))$

*Bäuerle*

### 1. *Semantic values: the theory of extension and intension*

Frege (1892), Carnap (1947)

*Rough characterisation*

- Every expression has (at least) two semantic values, its extension and its intension.
- Extension and intension respectively capture two communicative functions of linguistic meaning: reference and content.
- The intension of an expression comprises its possible extensions, its extension is relative to a point of reference in Logical Space.
- Logical Space consists of fully determinate situations and/or possible worlds.
- The point in Logical Space on which the extension depends is a parameter that may be represented by a variable.
- Extensions (and intensions) may depend on further, *contextual* parameters.
- Every intension may serve as an extension. => hierarchy of senses

*Two versions*

- **Strong:** Extensions are (stand-ins for) referents; intensions are (stand-ins for) content.

(3a)  $ext_w(Gottlob) = \text{Frege} = \rho_w(Gottlob)$

referent

(b)  $ext_w(Bertie) = \{A \subseteq D \mid \text{Russell} \in A\}; \rho_w(Bertie) = \iota(\cap(ext_w(Bertie)))$

stand-in

- **Weak:** Extensions help determining the referents of (possibly other, larger) expressions; intensions help determining the information values of (possibly other, larger) expressions.

(4a)  $int(earnest) = ext_w(\text{the property of being earnest}) = P$

internal value

(b)  $\rho_w(\text{the property of being earnest}) = P; \text{earnest} \notin \text{dom}(\rho_w)$

external value

### 2. *Indirect interpretation*

... with parameters (IL)

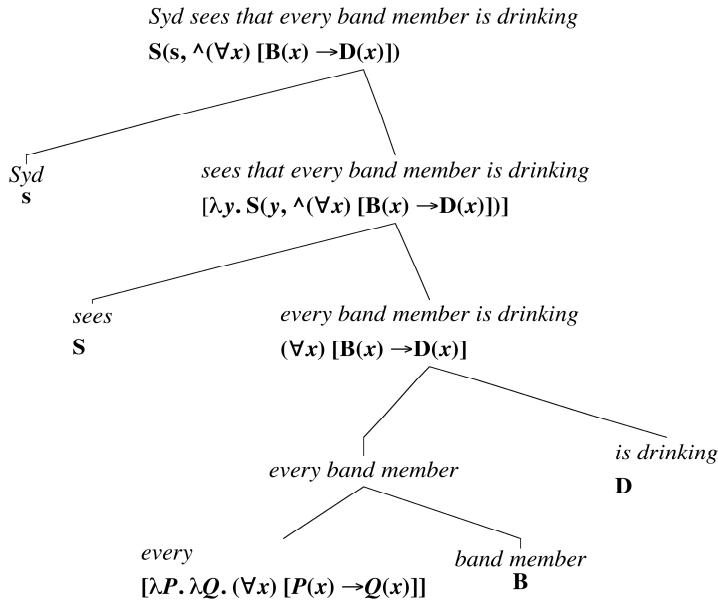
Montague (1970)

$\mid \text{earnest} \mid = \mathbf{E} \in \text{Con}_{e t}$

*Characteristics*

- Translations of expressions denote extensions relative to points in Logical Space.
- Intensions can be derived by functional abstraction from these points of reference expressed by the cap operator ' $\wedge$ ':
- The domain of denotation includes extensions of all types: truth values (type  $t$ ), individuals ( $e$ ), functional extensions ( $ab$ ) assigning extensions (of types  $a$ ) to extensions (of types  $b$ ), as well as intensions ( $sa$ ) – but not the points of Logical Space, or any functions into Logical Space.

(3)



... with variables (Ty2)

$| \textit{earnest} | = \mathbf{E}(i)$ , where  $\mathbf{E} \in \text{Con}_s(et)$

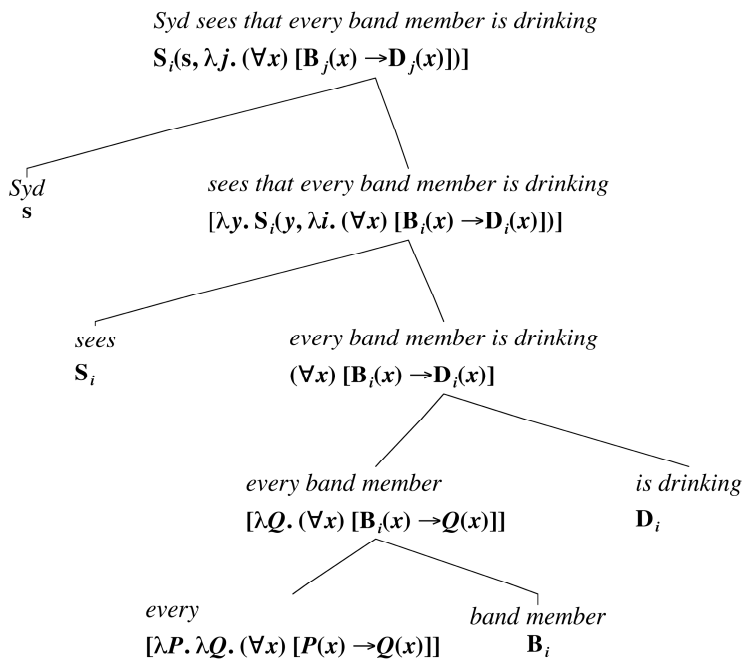
$\llbracket \mathbf{E}(i) \rrbracket^{g, \dots} = P(g(i)) \subseteq D$

Groenendijk & Stokhof (1982)

### Characteristics

- Translations of expressions are open formulae with a free variable  $i \in \text{Var}_s$  denoting the point of reference.
- Intensions can be derived by applying  $\lambda$ -abstraction to this variable.
- Apart from the extensions, the domain of denotation includes objects of *surplus* types: points of Logical Space ( $s$ ), and functions from denotations to denotations (e.g.,  $(se)s$ ).

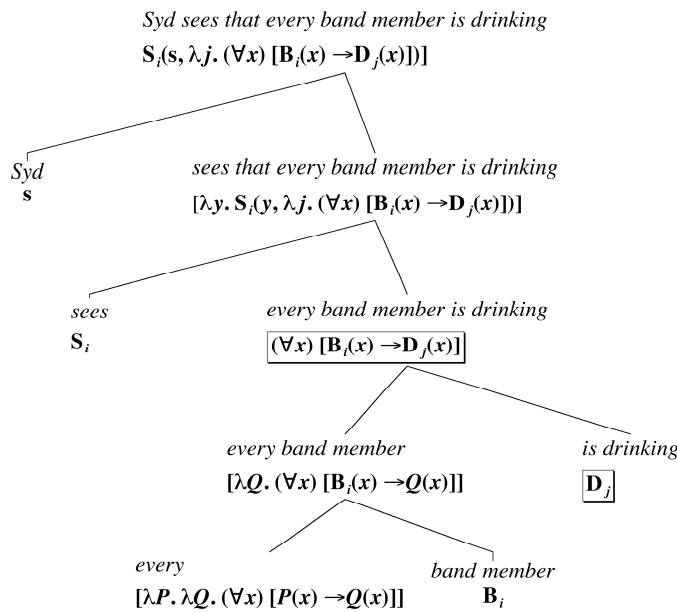
(4)



Comparison

- Any *IL*-formula  $\alpha$  may be expressed by a *Ty2*-formula  $\alpha^*$ , in which the fixed *Ty2*-variable  $i$  represents the point of reference.
- Not every *Ty2*-formula  $\beta$  is (equivalent to) some  $\alpha^*$ . In particular, (a)  $\beta$  may contain free variables of type  $s$  other than  $i$ ; (b)  $\beta$  may contain free variables or constants of surplus types; (c)  $\beta$  may itself be of a surplus type.
- Formulae  $\beta$  that satisfy one of (a)–(c) do not denote extensions and are thus *irrelevant* to the theory of extension and intension.
- All relevant *Ty2*-formulae can be expressed in *IL*. Zimmermann (1989)
- A *Ty2*-translation conforms to the theory of extension and intension iff it is equivalent to an *IL*-formula.

(5)



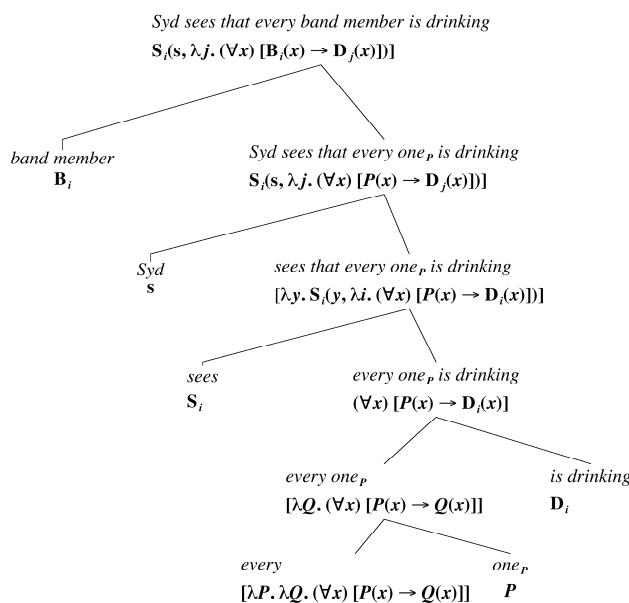
uninterpretable values?

Zimmermann (2012)

Restrictors as res

Groenendijk & Stokhof (1982)

(6)



### 3. A new approach to Bäuerle's problem

- (7a) Norman hears that Syd sees that every band member is drinking.  
 (b)  $\mathbf{H}^0(\mathbf{n}, \wedge \mathbf{S}^1(\mathbf{s}, \wedge \wedge (\forall x) [\mathbf{B}(x) \rightarrow \mathbf{D}(x)]))$  Frege (1892); cf. Parsons (1981)  
 (c)  $\mathbf{H}_i^0(\mathbf{n}, \lambda i. \mathbf{S}_i^1(\mathbf{s}, \lambda i. \lambda k. (\forall x) [\mathbf{B}_i(x) \rightarrow \mathbf{D}_i(x)]))$   
 $\equiv \mathbf{H}_i^0(\mathbf{n}, \lambda j. \mathbf{S}_j^1(\mathbf{s}, \lambda j. \lambda k. (\forall x) [\mathbf{B}_k(x) \rightarrow \mathbf{D}_k(x)]))$   
 $\equiv \mathbf{H}_i^0(\mathbf{n}, \lambda j. \mathbf{S}_j^1(\mathbf{s}, \lambda j. \lambda k. \forall (\mathbf{B}_k)(\mathbf{D}_k)))$   
 $\equiv \mathbf{H}_i^0(\mathbf{n}, \lambda j. \mathbf{S}_j^1(\mathbf{s}, \lambda j. \lambda k. [\lambda j. \lambda k. \forall (\mathbf{B}_k)](j)(k) [\lambda j. \lambda k. \forall (\mathbf{B}_k)](j)(k)])$   
 (d)  $\mathbf{H}_i^0(\mathbf{n}, \lambda j. \mathbf{S}_j^1(\mathbf{s}, \lambda j. \lambda k. [\lambda j. \lambda k. \forall (\mathbf{B}_j)](j)(k) [\lambda j. \lambda k. \forall (\mathbf{B}_k)](j)(k)])$   
 $\equiv \mathbf{H}_i^0(\mathbf{n}, \lambda j. \mathbf{S}_j^1(\mathbf{s}, \lambda j. \lambda k. \forall (\mathbf{B}_j)(\mathbf{D}_k)))$   
 $\equiv \mathbf{H}_i^0(\mathbf{n}, \lambda j. \mathbf{S}_j^1(\mathbf{s}, \lambda j. \lambda k. (\forall x) [\mathbf{B}_j(x) \rightarrow \mathbf{D}_k(x)]))$

From rigid intensions ...

$$\llbracket \wedge^n \alpha \rrbracket^{w_0} (w_1) \dots (w_n) = \llbracket \alpha \rrbracket^{w_n}$$

... to twisted senses:

$$\llbracket \wedge_m^n \alpha \rrbracket^{w_0, s} (w_1) \dots (w_n) = \llbracket \alpha \rrbracket^{w_m, s}$$

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