Compositionality Problems and How to Solve Them

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1. Compositionality …

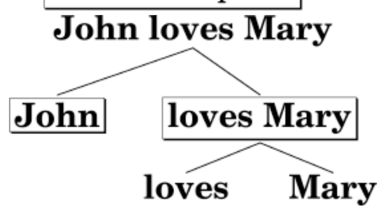
Generalised Principle of Compositionality cf. Zimmermann (2006; forthcoming)
The *V* of a complex expression functionally depends on the *V*s of its immediate parts and the way in which they are combined …where *V* is a semantic value

Ordinary Principle of Compositionality Montague (1970)
The meaning of a complex expression functionally depends on the meanings of its immediate parts and the way in which they are combined.

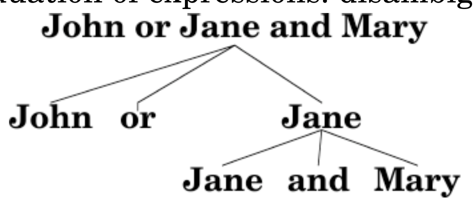
Extensional Principle of Compositionality Frege (1892)
The extension of a complex expression functionally depends on the extensions of its immediate parts and the way in which they are combined.

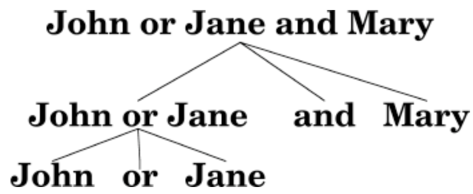
Intensional Principle of Compositionality Kaplan (1989)
The content of a complex expression functionally depends on the contents of its immediate parts and the way in which they are combined.

Parts vs. immediate parts

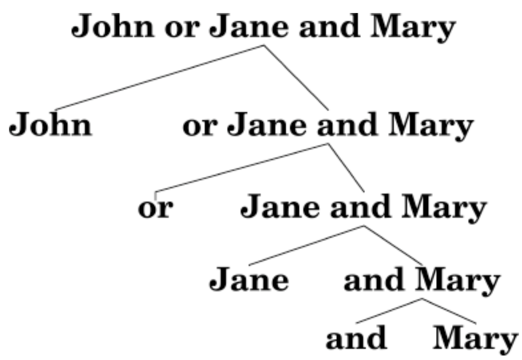


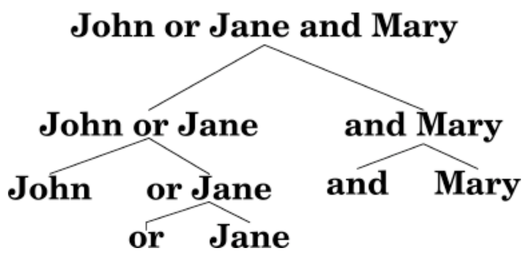
Individuation of expressions: disambiguation



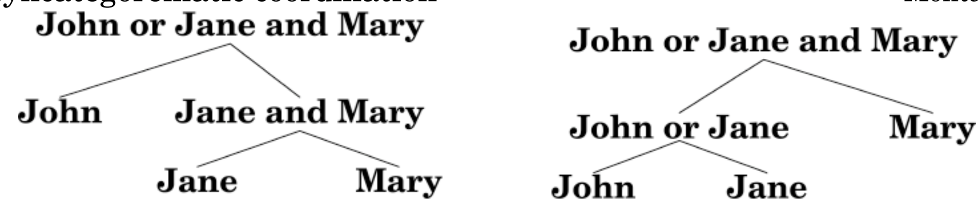


Simplification: binarity
… by coordination as modification

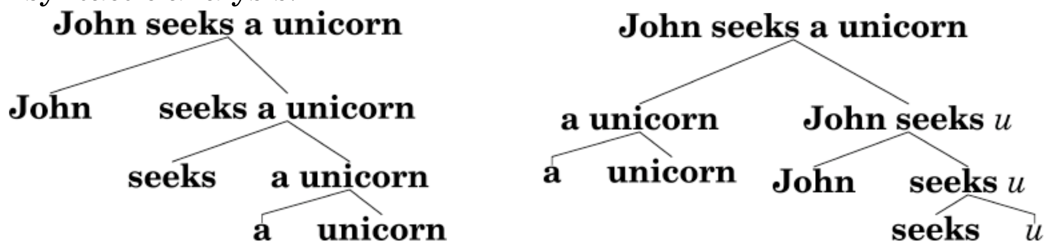




… or syncategorematic coordination Montague (1973)



Level of syntactic analysis:



Top-down strategy for determining semantic values: Frege (1884; 1892)

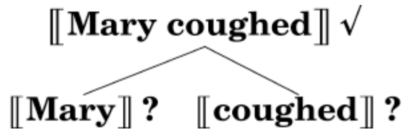
- Find suitable (‘cofinal’) set of expressions. Hodges (2001)
- Assign values to members.
- Fill in gaps applying suitable strategies.

2. Problems …

3 types of problems to be encountered in analysing expressions of the form

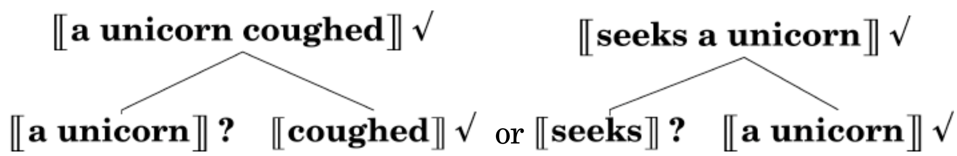
• *Type A*

GIVEN: value of whole
NEEDED: values of both parts
e.g.:



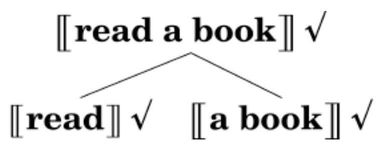
• *Type B*

GIVEN: value of whole and of one part
NEEDED: values of other part
e.g.:



• *Type C*

GIVEN: value of whole and of one part
NEEDED: combination of semantic values of parts
e.g.:



A compositionality problem is *solvable* just in case there is a way of replacing all **?** by √ without changing any √.

Observations cf. Zadrozny (1994), Hodges (2001)

Type *A* problems are always solvable.

A Type *B* problem is solvable iff

$[\![\boldsymbol{RIGHT}_i]\!] = [\![\boldsymbol{RIGHT}_j]\!]$ implies: $[\![\boldsymbol{WHOLE}_i]\!] = [\![\boldsymbol{WHOLE}_j]\!]$

[or: $[\![\boldsymbol{LEFT}_i]\!] = [\![\boldsymbol{LEFT}_j]\!]$ implies: $[\![\boldsymbol{WHOLE}_i]\!] = [\![\boldsymbol{WHOLE}_j]\!]$],

for all *i* and *j*.

A Type *C* problem is solvable iff

$[\![\boldsymbol{RIGHT}_i]\!] = [\![\boldsymbol{RIGHT}_j]\!]$ implies: $[\![\boldsymbol{WHOLE}_i]\!] = [\![\boldsymbol{WHOLE}_j]\!]$

and: $[\![\boldsymbol{LEFT}_i]\!] = [\![\boldsymbol{LEFT}_j]\!]$ implies: $[\![\boldsymbol{WHOLE}_i]\!] = [\![\boldsymbol{WHOLE}_j]\!]$],

for all *i* and *j*.

3. … and How to Solve Them

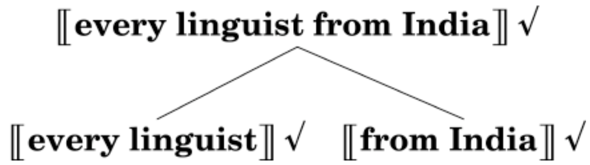
General Strategies for Unsolvable (and Solvable) Compositionality Problems

• Redefine syntactic input.

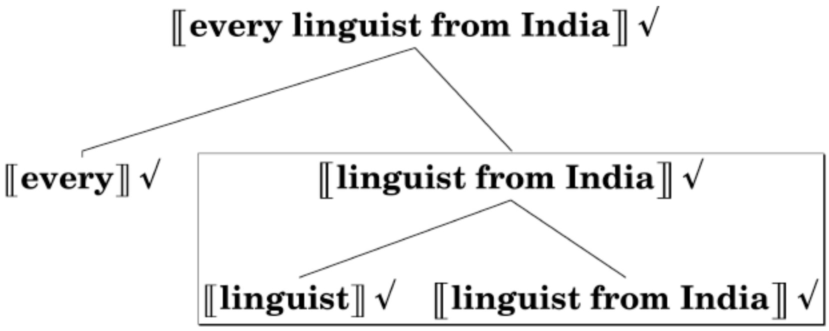
Applications:

– Type *C* (unsolvable), creating another, *solvable* Type *C* problem:

From:

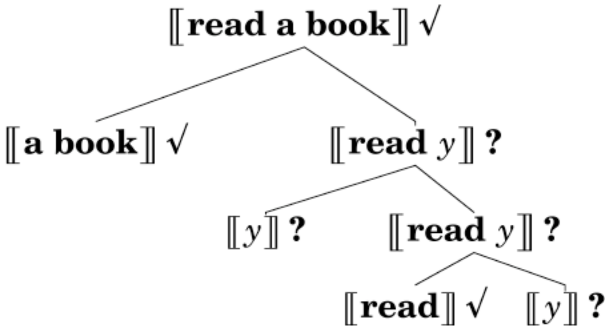


to:



– Type *C* (solvable), but creating more *Type B* and *Type C* problems…

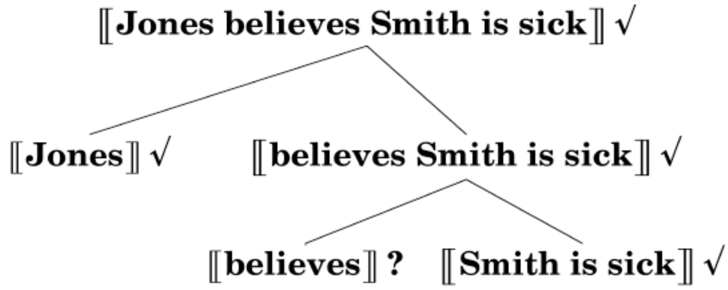
May (1985), Heim & Kratzer (1998)



• Replace semantic values by more fine-grained ones:
Type *B* (unsolvable):

If ⟦***X***⟧ is ***X***'s extension, then: Frege (1892)

(*)



is unsolvable. Replacing ⟦**Smith is sick**⟧ by the intension ‖**Smith is sick**‖ renders (*) solvable.

General Strategies for Solvabe Compositionality Problems

• <u>*Strategy A:*</u> Frege (1884)

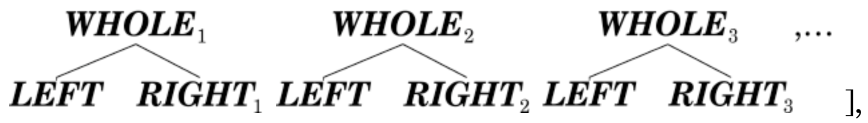
Find covariation between one part and some other entity, and take the latter to be the former's semantic value.

More precisely, given

(L)

[or:

(R)



],

find objects x_i such that:

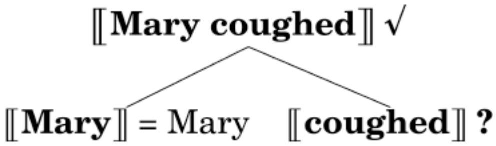
$$[\![\boldsymbol{WHOLE}_i]\!] = [\![\boldsymbol{WHOLE}_j]\!] \text{ just in case } [\![\boldsymbol{LEFT}_i]\!] = [\![\boldsymbol{LEFT}_j]\!]$$

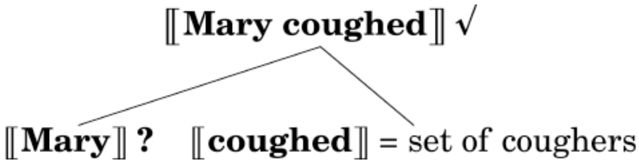
[or $[\![\boldsymbol{WHOLE}_i]\!] = [\![\boldsymbol{WHOLE}_j]\!]$ just in case $[\![\boldsymbol{RIGHT}_i]\!] = [\![\boldsymbol{RIGHT}_j]\!]$]

Then put:

$$[\![\boldsymbol{LEFT}_i]\!] := x_i \text{ [or } [\![\boldsymbol{RIGHT}_i]\!] := x_i\text{]}$$

Application:





• *Strategy B:* Frege (1892); cf. Kupffer (2008); Zimmermann (in prep.)

Determine primary occurrences of other expressions and construct their values as contributions in primary occurrences. More precisely, given

$[\![\boldsymbol{RIGHT}_1]\!]$, $[\![\boldsymbol{RIGHT}_2]\!]$,… and $[\![\boldsymbol{WHOLE}_1]\!]$, $[\![\boldsymbol{WHOLE}_2]\!]$,… construct:

$[\![\boldsymbol{RIGHT}_1]\!]$	$[\![\boldsymbol{WHOLE}_1]\!]$
$[\![\boldsymbol{RIGHT}_2]\!]$	$[\![\boldsymbol{WHOLE}_2]\!]$
…	…
$[\![\boldsymbol{RIGHT}_i]\!]$	$[\![\boldsymbol{WHOLE}_i]\!]$
…	…

[or:

$[\![\boldsymbol{LEFT}_1]\!]$	$[\![\boldsymbol{WHOLE}_1]\!]$
$[\![\boldsymbol{LEFT}_2]\!]$	$[\![\boldsymbol{WHOLE}_2]\!]$
…	…
$[\![\boldsymbol{LEFT}_i]\!]$	$[\![\boldsymbol{WHOLE}_i]\!]$
…	…

]

and put $[\![\boldsymbol{LEFT}]\!] := f$ such that:

$f([\![\boldsymbol{RIGHT}_i]\!]) = [\![\boldsymbol{WHOLE}_i]\!]$ [or $f([\![\boldsymbol{LEFT}_i]\!]) = [\![\boldsymbol{WHOLE}_i]\!]$]

Application:

$[\![\textbf{a unicorn coughed}]\!]$ √

$[\![\textbf{a unicorn}]\!] = f$ $[\![\textbf{coughed}]\!]$ √

where:

$f([\![\textbf{coughed}]\!]) = [\![\textbf{a unicorn coughed}]\!]$,

$f([\![\textbf{neighed}]\!]) = [\![\textbf{a unicorn neighed}]\!]$, etc.

If $[\![\boldsymbol{X}]\!]$ is $\boldsymbol{X}$'s extension, it turns out that:

$f = \lambda P.\vdash P \cap U \neq \emptyset \dashv$

$[\![\textbf{seeks a unicorn}]\!]$ √

$[\![\textbf{seeks}]\!] = f$ $[\![\textbf{a unicorn}]\!]$ √

where:

(*) $f([\![\textbf{a unicorn}]\!]) = [\![\textbf{seeks a unicorn}]\!]$

$f([\![\textbf{a horse}]\!]) = [\![\textbf{seeks a horse}]\!]$, etc.

If $[\![\boldsymbol{X}]\!]$ is $\boldsymbol{X}$'s extension, then:

$[\![\textbf{a unicorn}]\!] = [\![\textbf{a ghost}]\!]$

and so:

$f([\![\textbf{a unicorn}]\!]) = f([\![\textbf{a ghost}]\!])$

BUT: $f([\![\textbf{a unicorn}]\!]) = [\![\textbf{seeks a unicorn}]\!] = \{x \mid x \text{ seeks a unicorn}\}$

≠ $\{x \mid \text{x seeks a ghost}\} = [\![\textbf{seeks a ghost}]\!] = f([\![\textbf{a ghost}]\!])$

=> NO EXTENSIONAL SOLUTION!

If $[\![\boldsymbol{X}]\!]$ is $\boldsymbol{X}$'s intension, then:

$f([\![\textbf{a unicorn}]\!])(i) = \{x \mid x \text{ seeks a unicorn at [index] } i\}$

$f([\![\textbf{a horse}]\!])(i) = \{x \mid x \text{ seeks a horse at } i\}$

$f([\![\textbf{a ghost}]\!])(i) = \{x \mid x \text{ seeks a horse at } i\}$

unclear how (and even: whether) value depends on argument

=> additional theory needed cf. Zimmermann (2005: 2.3) [see Appendix]

• <u>*Strategy C:*</u>
Define combination F by collecting all instances:

$$F([\![\boldsymbol{LEFT}_i]\!], [\![\boldsymbol{RIGHT}_i]\!]) = [\![\boldsymbol{WHOLE}_i]\!]$$

and find pattern.
Application:

$$F([\![\boldsymbol{read}]\!], [\![\boldsymbol{a\ book}]\!]) = [\![\boldsymbol{read\ a\ book}]\!]$$

$$F([\![\boldsymbol{read}]\!], [\![\boldsymbol{every\ book}]\!]) = [\![\boldsymbol{read\ every\ book}]\!]$$

$$F([\![\boldsymbol{buy}]\!], [\![\boldsymbol{a\ book}]\!]) = [\![\boldsymbol{buy\ a\ book}]\!]$$

etc.
If $[\![\boldsymbol{X}]\!]$ is $\boldsymbol{X}$'s extension, we have:

$$F(\{(x,y) \mid x \text{ reads } y\}, \lambda P. \vdash P \cap \mathrm{B} \neq \emptyset \dashv) = \{x \mid \{y \mid x \text{ reads } y\} \cap \mathrm{B} \neq \emptyset\}$$

$$F(\{(x,y) \mid x \text{ reads } y\}, \lambda P. \vdash \mathrm{B} \subseteq \mathrm{P} \neq \emptyset \dashv) = \{x \mid \mathrm{B} \subseteq \{y \mid x \text{ reads } y\}\}$$

$$F(\{(x,y) \mid x \text{ buys } y\}, \lambda P. \vdash P \cap \mathrm{B} \neq \emptyset \dashv) = \{x \mid \{y \mid x \text{ buys } y\} \cap \mathrm{B} \neq \emptyset\}$$

etc.
– the pattern being:

$$F([\![\boldsymbol{LEFT}_i]\!], [\![\boldsymbol{RIGHT}_i]\!]) = [\![\boldsymbol{RIGHT}_i]\!](\{x \mid \{y \mid (x,y) \in [\![\boldsymbol{LEFT}_i]\!]\}\})$$

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Appendix: Excerpt from Zimmermann (2005: 220f.; fn. on notation omitted)

2.3. California. Summer of love. Montague, using techniques of higher-order modal logic, turned Quine's account of opacity into a surfacecompositional analysis[4], the starting point of which is a possible worlds adaptation of Quine's paraphrase formulated in intensional type theory; under the assumption that **try-for-it-to-be-the-case-that** denotes a binary relation between individuals and propositions, **seek a unicorn** receives the following logical analysis:[5]

(5) $\lambda x\, \boldsymbol{T}(x, (\exists y)[\boldsymbol{U}(y) \wedge \boldsymbol{F}(x, y)])$

Surface compositionality, i.e. a word-by-word analysis, is then achieved by applying the Fregean strategy of *meaning subtraction*[6], obtaining the meaning of **seek** by separating the quantifier expressed by **a unicorn** from the property denoted by **seek a unicorn**. Such a separation can be carried out using a series of lambda-abstractions:

$$
\begin{aligned}
& (5) \\
\equiv\; & \lambda x\, \boldsymbol{T}(x, \underline{[\lambda Q(\exists y)[\boldsymbol{U}(y) \wedge Q(y)]]}(\lambda y \boldsymbol{F}(x, y))) \\
\equiv\; & \underline{\underline{[\lambda \mathfrak{Q} \lambda x\, \boldsymbol{T}(x, \mathfrak{Q}(\lambda y \boldsymbol{F}(x, y)))]}}\underline{(\lambda Q(\exists y)[\boldsymbol{U}(y) \wedge Q(y)])}
\end{aligned}
$$

The first step isolates the (underlined) meaning of the indefinite **a unicorn** as contributing to the (varying) implicit attitude object denoted by **find a unicorn**; the second step separates that meaning from the rest, which in turn may serve as an analysis of **seek**. Given the types in (5), it turns out that **seek** denotes a relation between an individual and a quantifier, i.e. its meaning is of type $((et)t)(et)$, viz.:

(6) $\lambda \mathfrak{Q} \lambda x\, \boldsymbol{T}(x, (\mathfrak{Q} y) \boldsymbol{F}(x, y))$

where "$(\mathfrak{Q} y)\varphi$" abbreviates "$\mathfrak{Q}(\lambda y \varphi)$". Since the resulting type is independent of the paraphrase, Montague's analysis is more general than Quine's:

> As far as "seeks" and "owes" are concerned, circumlocution involving infinitives is possible. It is not, however, in the case of all English verbs sharing the logical peculiarities of "seeks" and "owes" [...]
>
> Montague (1969: 177)