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CHAPTER

10 Fregean Compositionality

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Abstract

Two distinctive features of Frege's approach to compositionality are reconstructed in terms of the theory of extension and intension: (i) its bias in favour of extensional operations; and (ii) its resort to indirect senses in the face of iterated opacity. While (i) has been preserved in current formal semantics, it proves to be stronger than a straightforward extensionality requirement in terms of Logical Space, the difference turning on a subtle distinction between extensions at particular points and extensions per se. (ii) has traditionally been dismissed as redundant, and is shown to lead to a mere 'baroque' reformulation of ordinary compositionality. Nevertheless, whatever Frege's motive, the very idea of having opaque denotations keep track of the depth of their embedding gives rise to a fresh view at certain scope paradoxes that had previously been argued to lie outside the reach of a binary distinction between extension and intension.

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The distinction between transparent and opaque contexts has always played a major rôle in theories of linguistic semantics, though it has undergone a number of reformulations and precisifications since its origins in Frege's (1892) classical substitution arguments. Most dramatically, the unfathomable distinction between *Sinn* and *Bedeutung* has been recast in more perspicuous, set-theoretic terms, trading Frege's senses for Carnap's (1947) intensions and identifying functions with their courses of values. Despite these transformations, a core part of the Fregean architecture has survived. In particular, (*i*) the strategy of treating extensionality as the default case of semantic composition and invoking intensions only when need be, has become part of most common approaches to the syntax-semantics interface. On the other hand, (*ii*) Frege's apparent commitment to a hierarchy of senses in the analysis of iterated opacity has been discarded for its alleged lack of cogency and coherence. The present chapter takes a closer look at both aspects of the Fregean architecture within the standard possible worlds framework of Montague (1970). Concerning (*i*), it will be argued that the Fregean strategy results in an interpretation of intensional constructions (i.e., opaque contexts) that goes beyond mere intensional compositionality in that it imposes a certain kind of uniformity on the pertinent semantic combinations. As to (*ii*), it will be shown how a hierarchy of intensions may help restore compositionality when extensional and intensional scope effects appear to be out of tune.

The historical background notwithstanding, the perspective in what follows will be a systematic one, aiming at a better understanding and possible improvement of compositionality in possible worlds semantics. Section 10.1 summarizes the theoretical background, essentially taken from Montague (1970), thereby fixing some terminology and notation. Section 10.2 addresses extensional compositionality and introduces the relevant notion of uniformity, drawing motivation from Frege's original treatment of transparent contexts. Section 10.3 adapts the uniformity criterion to intensional contexts and shows how the result strengthens pure intensional compositionality. Section 10.4 develops a redundant (and somewhat trivial) version of a hierarchy of intensions, which is modified in Section 10.5 so as to apply to intensional scope ↳ puzzles. The appendix contains a formalization of the previous two sections in terms of the intensional type logic of Montague (1970).

10.1 Background

For definiteness and ease of reference, the following considerations on compositionality are couched in terms of the general theoretical framework of Montague (1970), with some simplifications and (mostly terminological) deviations. In particular, the core of the syntax-semantics interface (of a given language) is taken to be a homomorphism from the algebra of syntactic *expressions* to the algebra of *meanings*.

The exact nature of the syntactic side will be of no concern, as long as it is understood that an expression may correspond to more than one surface form and that it uniquely splits up into its immediate parts (if any), which need not correspond to its surface parts. As a case in point, a (surface) sentence like (1) might be analysed as ambiguous between a specific and a non-specific reading, corresponding to expressions that disagree on whether the (surface) object is an immediate part (as in the specific reading) or not:

> (1) John seeks a unicorn.

Algebraically speaking, the expression (or structure) underlying the non-specific reading of (1) would then be the result of applying a syntactic operation $\mathscr{P}$ combining the (surface) subject and predicate (or, rather the expressions Δ_s and Δ_p underlying them), whereas the other reading will result from combining the object with the 'gappy' rest of the (surface) sentence by some other syntactic operation $\mathscr{Q}$;[1] thus $\mathscr{P}(\Delta_s, \Delta_p) \neq \mathscr{Q}(\Delta_o, \Delta_r)$, although both expressions surface as (1). Moreover, since the predicate is the result of combining the transitive verb with its direct object, it is also a complex syntactic object of the form $\Delta_p = \mathscr{O}(\Delta_v, \Delta_o)$ where $\mathscr{O}$ is a pertinent syntactic operation, Δ_v is the opaque (lexical) verb, and Δ_o is as before.

It will be convenient to assume that expressions are divided into syntactic categories, on which the syntactic operations have predictable effects. $\mathscr{P}$ and $\mathscr{Q}$ can thus be conceived of as two syntactic *constructions*, Predication and Quantification, which result in sentences when applied to expressions of suitable categories; similarly, Opaque Object Attachment $\mathscr{O}$ produces a predicate by combining an intensional transitive verb with an object. The formal and descriptive details of the syntactic expressions and their constructions are, however, largely irrelevant to what follows.

As to the semantic side, one may rely on the extensions and intensions defined in terms of the familiar hierarchy of functional types based on the sets D_t, D_e, and D_s of truth values, individuals, and possible worlds (or indices). In fact, for current p. 278 ↳ purposes the *intensional types* of Montague (1970), where ***s*** only marks functional domains (as opposed to ranges), turn out to suffice, thus ruling out types of the form (*a*,***s***) as well as ***s*** itself. Intensions, thus conceived, are functions of any types (***s***,***a***). In what follows, meanings may be identified with intensions, which then form the carrier set of the semantic algebra.[2] More specifically, objects of types *a* and (***s***,***a***) are employed as (potential) *extensions* and *intensions*, respectively.

The syntax-semantics homomorphism assigns to each expression Δ its intension $\llbracket\Delta\rrbracket$, thereby guaranteeing that the extensions of any expressions of the same category κ are of the same type τ_κ. In particular, τ_κ = ***t*** if κ is the category (or a subcategory) of declarative sentences. Consequently, the intensions of expressions of category κ are objects of type (***s***, τ_κ); in particular, the intension of a declarative sentence is a characteristic function of a set of indices that gives the truth conditions of that sentence. The aforementioned homomorphism is fully determined by (a) a lexical meaning assignment and (b) the meaning combinations corresponding to the syntactic constructions. The lexical meanings (a) are listed in the lexicon, as part of the interface. The operations (b) match the syntactic constructions in their -arities. (a) and (b) suffice to specify the meaning assignment precisely because it is assumed to be a homomorphism, which means that the following equation holds in general:

> (2) $\llbracket\mathscr{F}(\Delta_1,\ldots,\Delta_n)\rrbracket = \|\mathscr{F}\|(\llbracket\Delta_1\rrbracket,\ldots,\llbracket\Delta_n\rrbracket),$

where the expressions Δ_1, ..., Δ_n stand in the *n*-ary syntactic construction $\mathscr{F}$, which is interpreted by the semantic operation $\|\mathscr{F}\|$. Asa case in point, the predication construction $\mathscr{P}$ mentioned above is traditionally interpreted as *functional application*:[3]

> (3) $\|\mathscr{P}\|(x,P)(i) = P_i(x_i) \overset{\text{def}}{=\!=} P(i)(x(i))$

where *x* and *P* are objects of types (***s***, ***e***) and (***s***, (***e***, ***t***)), respectively, and $i \in D_s$. According to (3), $\|\mathscr{P}\|$ maps an individual concept and a property to (the characteristic function of) the set of indices at which the individual falling under the concept has the property. Applying (2) and (3) to the unspecific reading of (1) thus yields the familiar truth conditions:

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$$\text{(4)}\quad\begin{array}{lr}
\llbracket\mathscr{P}(\Delta_{John},\Delta_{seeks\,a\,unicorn})\rrbracket^{i} = 1 & \\
\text{iff}\ \ \llbracket\mathscr{P}(\Delta_{John},\Delta_{seeks\,a\,unicorn})\rrbracket(i) = 1 & \text{notation}: \llbracket\Delta\rrbracket^{i} \overset{\text{def}}{=} \llbracket\Delta\rrbracket(i) \\
\text{iff}\ \|\mathscr{P}\|(\llbracket\Delta_{John}\rrbracket, \llbracket\Delta_{seeks\,a\,unicorn}\rrbracket)(i) = 1 & \text{by (2)} \\
\text{iff}\ \ \llbracket\Delta_{seeks\,a\,unicorn}\rrbracket(i)(\llbracket\Delta_{John}\rrbracket(i)) = 1 & \text{by (3)} \\
\text{iff}\ \ \llbracket\Delta_{seeks\,a\,unicorn}\rrbracket^{i}(\llbracket\Delta_{John}\rrbracket^{i}) = 1 & \text{notation}
\end{array}$$

According to (4), the relevant reading of (1) is true at (or *of*)a point *i* just in case the extension of its subject is in the extension of its predicate (taken asa set). The predicate itself is obtained by feeding the verb its object, the semantic effect of which may be captured by the following operation:[4]

> (5) $\|\mathscr{O}\|(R,Q)(i) = R_i(Q)$

where *R* and *Q* are intensions of types (***s***, ((***s***, ((***e***, ***t***), ***t***)), (***e***, ***t***))) and (***s***, ((***e***, ***t***), ***t***)), respectively, and $i \in D_s$. According to (5), $\|\mathscr{O}\|$ maps a relation between individuals and second-order properties (i.e, properties of sets of individuals) and a second-order property to the property of standing in the relation to the second-order property. Applying (5) to the unspecific reading of (1) thus specifies the truth conditions in terms of the meanings of its subject, main verb, and object:

$$\text{(6)}\quad\begin{array}{lr}
\llbracket\mathscr{P}(\Delta_{John},\Delta_{seeks\,a\,unicorn})\rrbracket^{i} = 1 & \\
\text{iff}\ \ \llbracket\Delta_{seeks\,a\,unicorn}\rrbracket(i)(\llbracket\Delta_{John}\rrbracket(i)) = 1 & (4) \\
\text{iff}\ \ \llbracket\mathscr{O}(\Delta_{seeks},\Delta_{aunicorn})\rrbracket(i)(\llbracket\Delta_{John}\rrbracket(i)) = 1 & \text{def. } \Delta_{seeks\,a\,unicorn} \\
\text{iff}\ \|\mathscr{O}\|(\llbracket\Delta_{seeks}\rrbracket, \llbracket\Delta_{aunicorn}\rrbracket)(i)(\llbracket\Delta_{John}\rrbracket(i)) = 1 & (2) \\
\text{iff}\ \ \llbracket\Delta_{seeks}\rrbracket(i)(\llbracket\Delta_{aunicorn}\rrbracket)(\llbracket\Delta_{John}\rrbracket(i)) = 1 & (5) \\
\text{iff}\ \ \llbracket\Delta_{seeks}\rrbracket^{i}(\llbracket\Delta_{John}\rrbracket^{i}, \llbracket\Delta_{aunicorn}\rrbracket) = 1 & \text{notation}
\end{array}$$

According to (6), the relevant reading of (1) is true of a point *i* just in case the ordered pair formed by the extension of its subject and the intension of its object is in the extension of its predicate (taken as a binary relation). These illustrations should suffice as a background for the following general discussion.

10.2 Uniform Extensionality

There is an obvious similarity between the operations $\|\mathscr{P}\|$ and $\|\mathscr{O}\|$ from Section 10.1: they both match functions and arguments with the corresponding values. However, there are also some obvious differences. To begin with, the functions and arguments operated on come in reverse order, they are of different types, and so are the resulting values. More importantly though, the two operations disagree on the rôle of Logical Space in determining the functions, arguments, and values to p. 280 ↳ be matched. The difference becomes apparent when $\|\mathscr{P}\|$ and $\|\mathscr{O}\|$ are applied to determine the extensions of expressions constructed by $\mathscr{P}$ and $\mathscr{O}$. In both cases the general homomorphism schema (2) yields the intension, which in turn can be applied to a given point *i* to obtain the extension (at *i*). In the case of an expression of the form $\mathscr{P}(\Delta_1,\Delta_2)$, the extension is the result of applying the extension of Δ_2 to that of Δ_1:

> (7) $\llbracket\mathscr{P}(\Delta_1,\Delta_2)\rrbracket^{i} = \rrbracket\,\Delta_2\rrbracket^{i}(\llbracket\Delta_1\rrbracket^{i})$

This follows immediately from the definition (3) of “$\|P\|$” and the schema (2). In other words, the extension of such an expression (at a point *i*) can be obtained by suitably combining the extensions of its immediate parts (at *i*). An inspection of the corresponding equation for expressions of the form $\mathscr{O}(\Delta_1,\Delta_2)$ reveals that their extensions do not allow for such a pointwise calculation:

> (8) $\llbracket\mathscr{O}(\Delta_1,\Delta_2)\rrbracket^{i} = \llbracket\Delta_1\rrbracket^{i}(\llbracket\Delta_2\rrbracket)$

The details of combinations like (8) will be addressed in Section 10.3. For the time being, it suffices to note that they do not satisfy the following criterion:

> (9) *Definition*
>
> A construction $\mathscr{F}$ (of *n* places) is *extensional* iff, at any point $i \in D_s$, the extension of an expression of the form $\mathscr{F}(\Delta_1,\ldots,\Delta_n)$ at *i*, is determined by the extensions of its immediate parts at *i*, in other words:
>
> - $\llbracket\mathscr{F}(\Delta_1,\ldots,\Delta_n)\rrbracket^{i} = \llbracket\mathscr{F}(\Delta'_1,\ldots,\Delta'_n)\rrbracket^{i}$
>   whenever $\llbracket\Delta_1\rrbracket^{i} = \llbracket\Delta'_1\rrbracket^{i},\ldots,\llbracket\Delta_n\rrbracket^{i} = \llbracket\Delta'_n\rrbracket^{i}$
>
> for any (appropriate) expressions $\Delta_1,\Delta'_1,\ldots,\Delta_n,\Delta'_n$.

The criterion defined in (9) relates to expressions and their interpretations; but it may be reformulated in terms of intensions and semantic operations alone:

> (10) *Definition*
>
> A semantic operation $\|\mathscr{F}\|$ is *extensional* iff, for any $i \in D_s$:
>
> - $\|\mathscr{F}\|(x_1, \ldots, x_n)(i) = \|\mathscr{F}\|(x'_1, \ldots, x'_n)(i)$
>   whenever $x_1(i) = x'_1(i), \ldots, x_n(i) = x'_n(i)$
>
> for any (appropriate) intensions $x_1, x'_1, \ldots, x_n, x'_n$[5]

Given (2), (10) easily implies (9) in the sense that $\mathscr{F}$ is extensional as soon as $\|\mathscr{F}\|$ is. The reverse does not hold, since the criterion in (10) may be violated by ineffable intensions. Still, p. 281 (10) is the more straightforward formulation, given that semantic operations are usually defined by extrapolating from intensions of expressions to ↳ intensions *tout court*.[6] It is readily seen that (9) and (10) amount to the following extensionality criteria:

> (11) *Facts*
>
> a. $\mathscr{F}$ is extensional iff for any $i \in D_s$ there is an operation $\|\mathscr{F}\|_i$ such that
>
> - $[\![\mathscr{F}(\Delta_1, \ldots, \Delta_n)]\!]^{\,i} = \|\mathscr{F}\|_i([\![\Delta_1]\!]^{\,i}, \ldots, [\![\Delta_n]\!]^{\,i})$
>
> for any (appropriate) expressions $\Delta_1, \ldots, \Delta_n$.
>
> b. $\|\mathscr{F}\|$ is extensional iff for any $i \in D_s$ there is an operation $\|\mathscr{F}\|_i$ such that
>
> - $[\![\mathscr{F}(x_1, \ldots, x_n)]\!]^{\,i} = \|\mathscr{F}\|_i(x_1(i), \ldots, x_n(i))$
>
> for any (appropriate) intensions $x_1, \ldots, x_n$.

It should be noted that, the $\|\mathscr{F}\|_i$ in (11a) and (11b) operate on extensions rather than intensions. Moreover, each of them is unique (relative to *i*) if it exists, so that the equation should be taken as a definition of the notation: given any *i*, $\|\mathscr{F}\|_i$ is *that* operation on extensions (of expressions of the relevant types) that satisfies the relevant equation. We are using the same notation for them since the difference between the two is immaterial to our concerns.

In the above definitions, the extensions of the daughter expressions must combine to produce the extensions of their mothers. This characterization may give rise to the impression that extensional constructions, thus conceived, are natural descendants of Frege's (1892) transparent contexts (or *gerade Rede*). However, the notion of compositionality defined in (10), straightforward though it may seem from a Carnapian stance, is a far cry from its Fregean origins. The reason is that extensional constructions, rather than just combining extensions as such, combine extensions of expressions *qua* being extensions at a given point in Logical Space. This is so because the corresponding operations $\|\mathscr{F}\|_i$ on extensions depend on the particular point *i* in focus—as is brought out in the notation used in (11). Obviously an operation that combines extensions independently of the particular point at which they are determined, needs to meet the following, stricter condition:

> (12) *Definition*
>
> A semantic operation $\|\mathscr{F}\|$ is *uniformly extensional* iff there is an operation $\|\mathscr{F}\|_*$ such that, for any $i \in D_s$:
>
> - $\|\mathscr{F}\|(x_1, \ldots, x_n)(i) = \|\mathscr{F}_*(x_1(i), \ldots, x_n(i))$
>
> for any (appropriate) intensions $x_1, \ldots, x_n$, and $x'_1, \ldots, x'_n$.

p. 282 A similar uniformity criterion for syntactic operations could be formulated along the lines of (9); this is left for the reader. Clearly, (12) implies (10), but it is stronger: in order to be extensional, an operation needs to combine extensions at every point; in order to be uniformly extensional, it must always combine them in the same way. The following highly idiosyncratic binary coordination construction $F^{\#}$ illustrates the difference:[7]

> (13) $\left\|\mathscr{F}^{\#}\right\|(p, q)(i) = \begin{cases} min(p(i), q(i)), & \text{if } i \neq i^{\#} \\ max(p(i), q(i)), & \text{otherwise} \end{cases}$

In (13) *p* and *q* are arbitrary propositions of type (***s***, ***t***), and $i^{\#} \in D_s$ is some (arbitrary but fixed) point of reference—the best of all possible worlds, say. Hence $\mathscr{F}^{\#}$ works like Boolean conjunction throughout Logical Space—except at $i^{\#}$, where it boils down to disjunction. The very form of the equations in (13) reveals that $\|\mathscr{F}\|$ is extensional in the sense of (10): at any point, the extension of an $\mathscr{F}^{\#}$ -expression can be calculated from the extensions of its immediate parts, in other words: the truth values they have at that point; in the notation of (11), $\|\mathscr{F}\|_{i^{\#}} = max$, whereas $\|\mathscr{F}\|_i = min$, if $i \neq i^{\#}$. In view of this characterization, $\|\mathscr{F}\|$ clearly lacks uniformity.[8]

The construction $\mathscr{F}^{\#}$ is suspiciously artificial. To begin with, unlike interpretations of naturally occurring syntactic constructions, it is not of a purely combinatory nature, defying usual criteria of logicality like isomorphism-invariance or definability in (fragments of) type logic.[9] However, certain more sophisticated operations would reveal that this is an accidental defect. In fact, the following variation of (13) turns out to be both *Ty*2-definable (and thus invariant) and extensional in the sense of (10), while still failing the uniformity criterion (12):[10]

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> (14) $\left\|\mathscr{F}^{\#\#}\right\|(\varphi, \psi)(i)(j) = \begin{cases} max(\varphi(i)(j), \psi(i)(j)), & \text{if } \varphi(i)(i) = \psi(i)(i) = 1 \\ min(\varphi(i)(j), \psi(i)(j)), & \text{otherwise} \end{cases}$

$\|\mathscr{F}^{\#\#}\|$ combines two *propositional concepts* $\varphi$and ψ of type (***s***, (***s***, ***t***)); hence the extensions of the syntactic construction $\mathscr{F}^{\#\#}$ would have to be propositions, as in the partition semantics of interrogative clauses.[11] Like $\|\mathscr{F}^{\#}\|$, $\|\mathscr{F}^{\#\#}\|$ shifts between two Boolean operations, viz. union and intersection on subsets of Logical Space (as represented by their characteristic functions). And again, it may intersect in one world and unite in another one, even though the daughter constituents express the same propositions. So at each point they combine extensions, but at different points they may combine them in different ways. To determine the extensions of expressions constructed by $\|\mathscr{F}^{\#}\|$ or $\|\mathscr{F}^{\#\#}\|$, then, it is not enough to know what the extensions of their immediate parts are; one also needs to know at which point these extensions are to be combined.

What is odd about both $\|\mathscr{F}^{\#}\|$ and $\|\mathscr{F}^{\#\#}\|$ is the very fact that they are not uniform. In fact, it would seem that extensionality never occurs without uniformity.[12] After all, typically extensional constructions like $\mathscr{P}$[redication] do not display any variation across Logical Space: $\|\mathscr{P}\|_i = \|\mathscr{P}\|_j$, for any points $i, j \in D_s$. Semantically, an extensional construction is a way of combining extensions, in other words, an operation on extensions—and just that: given an extensional construction (restrictive relative clause attachment, say), all it takes to predict its extension is the extensions of the parts so construed; in particular, it is immaterial what the point of reference is at which the parts have the extensions to be operated on. In other words, extensional constructions are not merely extensional; they are uniformly extensional. They do not just combine the extensions that expressions happen to have at given points of reference; they combine them no matter what point of reference is at stake.

The difference between extensionality *tout court* and uniform extensionality brings to the fore a lacuna in the popular (and loose) formulation of the principle of extensional compositionality:

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- ↳ The extension of a complex expression is determined by the extensions of its (immediate) parts and the way they are combined.

Since extensions, by definition, depend on Logical Space, a slightly more accurate formulation would be:

- At any point of reference *i*, the extension of a complex expression *at i* is determined by the extensions of its (immediate) parts *at i* and the way they are combined.

This reformulation brings out that extensionality does not in itself guarantee uniformity: the dependence may, after all, vary across Logical Space. However, no such variation is even conceivable if extensions are thought of as objects *per se*, as in the following, admittedly more cumbersome formulation of a principle of uniform extensional compositionality, which is meant to bring out that the combination is blind to the particular point of reference:

- At any point of reference *i*, the extension of a complex expression *at i* is determined by *the objects that happen to be the extensions of its* (*immediate*) *parts at i* and the way they are combined.

Since on Frege's (1892) theory of *Sinn* and *Bedeutung*, the dependence of extensions (or their combinations) on Logical Space does not even enter the picture, uniform extensionality certainly better matches Fregean transparency than does extensionality *tout court*.[13] The point is brought out particularly clearly in the arguably Fregean framework of *type-driven interpretation*, where extensional composition is determined by the constituents' extension types, which do not vary across Logical Space.[14]

10.3 Selective Extensionality

Extensional compositionality only holds for limited fragments of natural language. As Frege (1892) famously observed, not all constructions are extensional; for some resist extensional substitution *salva extensione* and can thus not be interpreted in terms of operations on extensions. The opaque object construction $\mathscr{O}$ is a case in point. The predicates in (15a) and (15b) may have different extensions (and the sentences formed by them different truth values), even though their objects may not:

> (15)
>
> a. seeks [a French restaurant that serves *bouillabaisse*]$_1$
>
> b. seeks [a French restaurant that serves *ratatouille*]$_2$
>
> p. 285 ↳ As a consequence, $\|\mathscr{O}\|$ does not meet the extensionality criterion (9); otherwise, the following chain of equations would have to hold for certain *i* at which (!) $\llbracket\Delta_1\rrbracket^{\,i} = \llbracket\Delta_2\rrbracket^{\,i}$

> (16) $$\llbracket\Delta_{(15a)}\rrbracket^{\,i}\,\underline{\underline{11a}}\left\|\mathscr{O}\right\|_i\left(\llbracket\Delta_{seeks}\rrbracket^{\,i},\ \llbracket\Delta_1\rrbracket^{\,i}\right)$$ $$\underline{\underline{(!)}}\quad\left\|\mathscr{O}\right\|_i(\llbracket\Delta_{seeks}\rrbracket^{\,i},\ \llbracket\Delta_2\rrbracket^{\,i})\underline{\underline{11a}}\ \llbracket\Delta_{(15b)}\rrbracket^{\,i}$$

where $\|\mathscr{O}\|_i$ is constructed according to (11). Frege himself did not consider the particular construction $\mathscr{O}$ but made the same point about clausal embedding $\mathscr{A}$ under attitude verbs, which will also be the prime example for the rest of this chapter:

> (17)
>
> a. thinks [that] Mary is sick
>
> b. thinks [that] 2+2=5

Obviously, the embedded clauses may have the same truth values while the predicates in (17) have different extensions. Again, for no *i* at which (!!) the embedded clauses in (17) are materially equivalent, can there be an operation $\|\mathscr{A}\|_i$ satisfying (11); for otherwise (18) would hold:

> (18) $$\llbracket\Delta_{(17a)}\rrbracket^{\,i}\overset{(11a)}{=}\left\|\mathscr{A}\right\|_i\left(\llbracket\Delta_{thinks}\rrbracket^{\,i},\ \llbracket\Delta_{Mary\,is\,sick}\rrbracket^{\,i}\right)$$ $$\overset{(!!)}{=}\left\|\mathscr{A}\right\|_i\left(\llbracket\Delta_{thinks}\rrbracket^{\,i},\ \llbracket\Delta_{2+2=5}\rrbracket^{\,i}\right)\overset{(11a)}{=}\llbracket\Delta_{(17b)}\rrbracket^{\,i}$$

Given that some constructions, like $\mathscr{O}$ and *A*, fail to be extensional, the general format for specifying the interpretation of (binary) constructions cannot be (19a), let alone the uniform version (19b):

> (19)
>
> a. $\llbracket\mathscr{F}(\Delta_1,\Delta_2)\rrbracket^{\,i} = \|\mathscr{F}\|_i(\llbracket\Delta_1\rrbracket^{\,i},\ \llbracket\Delta_2\rrbracket^{\,i})$
>
> b. $\llbracket\mathscr{F}(\Delta_1,\Delta_2)\rrbracket^{\,i} = \|\mathscr{F}\|_*(\llbracket\Delta_1\rrbracket^{\,i},\ \llbracket\Delta_2\rrbracket^{\,i})$

The failure of (19a) may suggest that compositionality in general only works on the intensional level and therefore calls for equations like (19c), which is merely the binary version of the most general (intensional) compositionality schema (2) above:

> (19)
>
> c. $\llbracket\mathscr{F}(\Delta_1,\Delta_2)\rrbracket = \|\mathscr{F}\|(\llbracket\Delta_1\rrbracket,\ \llbracket\Delta_2\rrbracket)$

To get (19c) closer to (19a) and (19b), the equation can be rewritten in a pointwise fashion,[15] obtaining each maternal extension from the filial intensions:

> (19)
>
> c′. $\llbracket\mathscr{F}(\Delta_1,\Delta_2)\rrbracket = \|\mathscr{F}\|_i(\llbracket\Delta_1\rrbracket,\ \llbracket\Delta_2\rrbracket)$
>
> However, constructions like $\mathscr{O}$ and $\mathscr{A}$ are no reason to resort to equations like (19c) and (19c′), which rely on the full power of intensional compositionality. For although they cannot be interpreted as combinations of extensions, they need not be interpreted as combinations of intensions either: as far as their first argument is concerned, they are perfectly extensional, in the following, *selective* sense:

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> > ↳ (20) *Definitions*
> >
> > a. An n-place semantic operation is *k-extensional* (where $1 \leq k \leq n$) iff for any $i \in D_s$ and any (appropriate) intensions $x_1, \ldots, x_n$, and x'_k : if $x_k(i) = x'_k(i)$, then:
> >
> > - $\|\mathscr{F}\|\,(x_1,\ldots,x_k,\ldots,x_n)(i) = \|\mathscr{F}\|\,(x_1,\ldots,x'_k,\ldots,x_n)(i)$.
> >
> > b. $\|\mathscr{F}\|$ is *K-extensional* iff $K \subseteq \{k \mid k$ is k-extensional$\}$.
>
> Hence the extensional n-place operations in the sense of (10) are those that are *k*extensional for any $k \leq n$ and thus K-extensional, where $K = \{1, \ldots, n\}$; in particular, $\|\mathscr{P}\|$ is both 1-extensional and 2-extensional and thus {1, 2}-extensional. Moreover, observation (11b) easily adapts to the notions defined in (20):
>
> > (21) *Facts*
> >
> > a. $\|\mathscr{F}\|$ is k-extensional iff for any $i \in D_s$ there is an operation $\|\mathscr{F}\|_i$ such that, for any (appropriate) intensions $x_1, \ldots, x_n$:
> >
> > - $\|\mathscr{F}\|(x_1,\ldots,x_k,\ldots,x_n)(i) \ = \|\mathscr{F}\|_i(x_1,\ldots,x_k(i),\ldots,x_n)$.
> >
> > b. $\|\mathscr{F}\|$ is K-extensional iff for any $i \in D_s$ there is an operation $\|\mathscr{F}\|_i$ such that, for any (appropriate) intensions $x_1, \ldots, x_n$:
> >
> > - $\|\mathscr{F}\|(x_1,\ldots,x_k,\ldots,x_n)(i) \ = \|\mathscr{F}\|_i\,(x_1[i],\ldots,x_n[i])$,
> >
> > where (for any $k \leq n$):
> >
> > - $x_k[i] = \begin{cases} x_k(i), & \text{if } k \in K \\ x_k, & \text{if } k \notin K \end{cases}$

Clearly, neither $\|\mathscr{O}\|$ nor $\|\mathscr{A}\|$ are 2-extensional, though both are 1-extensional. Applying (21a) to $\|\mathscr{O}\|$ gives rise to the following equation:

> (22) $\|\mathscr{O}\|_i(\mathrm{R}, Q) = \mathrm{R}(Q)$,

where R and Q are of types ((**s**, ((**e**, **t**), **t**)), (**e**, **t**)) and (**s**, ((**e**, **t**), **t**)), respectively. The difference between (22) and the above equation (5) lies in the type of the first argument: where $\|\mathscr{O}\|$ operates on the intension of the opaque verb, $\|\mathscr{O}\|_i$ takes its extension (at point *i*). In a similar vein, one may use (21a) to reformulate the (straightforward) fully intensional interpretation (23) of $\mathscr{A}$ as in (24), where *S*, *p*, and S are of types (**s**, ((**s**, **t**), (**e**, **t**))), (**s**, **t**), and ((**s**, **t**), (**e**, **t**)), respectively:[16]

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> (23) $\|\mathscr{A}\|(S, p)(i) = S_i(p)$

> (24) $\|\mathscr{A}\|_i(\mathrm{S}, p) = \mathrm{S}(p)$

Two points are noteworthy about the reformulation (24). First of all, it shows that the extension of the predicate of an attitude report can be obtained by combining the extension of the verb and the intension of its complement, thus capturing the spirit of the general Fregean strategy of composing extensions whenever possible, invoking intensions only when need be, in other words, for those places *k* for which a construction is not *k*-extensional. Secondly, (24) reveals that the way in which the two semantic values are combined does not depend on the point *i* in Logical Space: just like the general definition (11) of extensionality, the characterization of *k*-extensionality does not impose any uniformity requirement. However, as in the case of extensional compositionality *tout court*, real-life selective extensionality seems to exist only in its uniform variety:[17]

> (25) *Definition*
>
> $\|\mathscr{F}\|$ is *uniformly K-extensional* iff $\|\mathscr{F}\|$ is *K*-extensional and there is an operation $\|\mathscr{F}\|_*$ such that, for any $i \in D_s$, $\|\mathscr{F}\|_* = \|\mathscr{F}\|_i$, as defined in (21b).

If $\mathscr{F}$ is a (syntactic) construction whose interpretation $[\![\mathscr{F}]\!]$ is uniformly *K*extensional, then its k^{th} argument position will be called *transparent* if $k \in K$, and *opaque* otherwise. The notation in (25) may be applied to the above token constructions:

> (26)
>
> a. $\|\mathscr{P}\|_*\,(\mathrm{x}, P) = P(\mathrm{x})$
>
> b. $\|\mathscr{O}\|_*(\mathrm{R}, Q) = \mathrm{R}(Q)$
>
> c. $\|\mathscr{A}\|_*(\mathrm{S}, p) = \mathrm{S}(p)$

Depending on whether they are italicized or not, the arguments of the semantic operations in (26) are understood to be suitable intensions or extensions, respectively. The operations may then be seen as combining the *compositionally relevant denotations*, which coincide with extensions in the case of transparent positions, and with intensions otherwise. The fact that the two cases match Frege's distinction between *gerade* and *ungerade* suggests that *Bedeutungen* in general should not be thought of as extensions but as compositional contributions. By (Fregean) default, the latter are extensions, as in (26a). But if a particular position fails to be extensional, mixed combinations as in (26b) and (26c) are invoked, trading extensions for intensions *in that position*; in Fregean parlance, these substitutes are *ungerade Bedeutungen* [≈ indirect meanings] of the expressions *in that position*.

p. 288 The 'local repair' strategy for opaque positions is not only in line with Frege's original approach. It has also been the part of standard formal semantics procedure ever since Montague (1970)—partly, no doubt, because it is more convenient than having to carry about intensions all the way through compositional derivations, and because notation and insights from extensional analysis immediately adapt to intensional constructions. More importantly, though, as the above considerations have brought out, in the face of substitutional failure the 'extensions-as-far-as-possible compositional rules' (to use Brian Rabern's (p.c.) term) do not merely boil down to intensional compositionality at large, but rather amount to adopting the slightly stronger principle of uniform selective extensionality as defined in (25).

10.4 The Hierarchy of Intensions

It thus seems as if uniform extensionality, if need be in its selective form, is an adequate reconstruction of Frege's basic compositional strategy within the theory of extension and intension; in any case it is in line with what is usually said and done in that tradition. However, there is at least one aspect in which this account diverges from Fregean compositional architecture, and that is the iteration of opacity. For according to Frege, the compositional contribution of an expression in an opaque position does not always coincide with its *ordinary* intension. Rather, this is only so if the relevant occurrence of the intensional construction is not itself part of an opaque position. Otherwise, in other words, if an expression in an opaque position is itself part of a larger opaque environment, its compositional contribution would have to be what the ordinary intension is to the extension it is supposed to replace; it would thus be an indirect intension (or *in-intension*), in other words, the ordinary intension as depending on Logical Space. Unless the larger opaque environment is itself part of a yet larger opaque environment in which case the contribution in question would have to be yet another step further away from the extension, in other words, the expression's in-in-intension, which would be its indirect intension as depending on Logical Space. And so on.[18] Whatever the rationale behind this move, it is not necessitated by the above Fregean strategy. For iteration of opacity can certainly be accounted for without it, and without further ado. Here is a case in point:

> (27)
>
> a. Norman hears that Syd sees that <u>Emily plays</u>.
>
> b. $\mathscr{P}\,(\Delta_{Norman}, \mathscr{A}\,(\Delta_{hears}, \mathscr{P}\,(\Delta_{Syd}, \mathscr{A}\,(\Delta_{sees}, \mathscr{P}\,(\Delta_{Emily}, \Delta_{plays})))))$

p. 289 Given the underlying structure (27b), the underlined clause in (27a) is in an opaque position that is itself part of a larger opaque environment. This is so because clausal embedding $\mathscr{A}$ is not 2-extensional; (the structure underlying) the clause occurs in that position; and (the relevant occurrence of) the construction itself is part of the larger expression

- $\mathscr{P}(\Delta_{Syd}, \mathscr{A}(\Delta_{sees}, \mathscr{P}(\Delta_{Emily}, \Delta_{plays})))$

occupying the 2nd argument position of (another occurrence of) *A*. Clearly, the general compositionality schema (2) does cover this constellation, to wit:

> (27)
>
> c. $$\begin{aligned}&[\![\mathscr{P}(\Delta_{Norman}, \mathscr{A}(\Delta_{hears}, \mathscr{P}(\Delta_{Syd}, \mathscr{A}(\Delta_{sees}, \mathscr{P}(\Delta_{Emily}, \Delta_{plays})))))]\!]\\ &= \|\mathscr{P}\|\,(\ [\![\Delta_{Norman}]\!]\ ,\ \|\mathscr{A}\|\|\,(\ [\![\Delta_{hears}]\!]\ , \|\mathscr{P}\|\,(\ [\![\Delta_{Syd}]\!]\ , \|\mathscr{A}\|\,(\ [\![\Delta_{sees}]\!]\ , \|\mathscr{P}\|\,(\ [\![\Delta_{Emily}]\!]\ ,\ [\![\Delta_{plays}]\!]\ )))))\\ &= \|\mathscr{P}\|_*(n, \|\mathscr{A}\|(H, \|\mathscr{P}\|(s, \|\mathscr{P}\|(S, \|\mathscr{P}\|(e, P)))))\end{aligned}$$

where the lexical intensions are given mnemonic abbreviations. And, clearly, if the general compositional setting can cope with double embedding, then so can the special case in which all operations are of the (selectively) extensional kind. In particular, the extension of (27a) at a point $i \in D_s$ can be determined following the above Fregean strategy:

> (27)
>
> d. $$\begin{aligned}&[\![\mathscr{P}(\Delta_{Norman}, \mathscr{A}(\Delta_{hears}, \mathscr{P}(\Delta_{Syd}, \mathscr{A}(\Delta_{sees}, \mathscr{P}(\Delta_{Emily}, \Delta_{plays})))))]\!]^{\,i}\\ &= \left\|\mathscr{P}\right\|_*\Big(\ [\![\Delta_{Norman}]\!]^{\,i}, \left\|\mathscr{A}\right\|_*\Big(\ [\![\Delta_{hears}]\!]^{\,i},\ \|\mathscr{P}\|\,(\ [\![\Delta_{Syd}]\!]\ , \|\mathscr{A}\|\,(\ [\![\Delta_{sees}]\!]\ , \|\mathscr{P}\|\,(\ [\![\Delta_{Emily}]\!]\ ,\ [\![\Delta_{plays}]\!]\ )))\Big)\Big)\\ &= \|\mathscr{P}\|_*(\mathrm{n}, \|\mathscr{A}\|_*(H_i, \|\mathscr{P}\|(s, \|\mathscr{A}\|(S, \|\mathscr{P}\|(e, P))))),\end{aligned}$$

Still, the calculation (27d) may feel slightly disappointing; for it does not introduce any of the $**$-operations on extensions in more deeply embedded positions, even where the latter are transparent. To make up for this, one can boost up semantic notation by borrowing λ-abstraction from type logic,[19] and expand (27d) by:

> (27)
>
> e. $$\begin{aligned}&[\![\mathscr{P}(\Delta_{Norman}, \mathscr{A}(\Delta_{hears}, \mathscr{P}(\Delta_{Syd}, \mathscr{A}(\Delta_{sees}, \mathscr{P}(\Delta_{Emily}, \Delta_{plays})))))\ ]\!]^{\,i}\\ &= \ldots\\ &= \|\mathscr{P}\|_*(\mathrm{n}, \|\mathscr{A}\|_*(H_i, \lambda j.\, \|\mathscr{P}\|_*(\mathrm{s}, \|\mathscr{A}\|_*(S_j, \lambda k.\, \|\mathscr{P}\|_*(\mathrm{e}, P_k)))))\\ &= H_i(\mathrm{n}, \lambda j.\, S_j(\mathrm{s}, \lambda k.\, P_k(\mathrm{e})))\end{aligned}$$

(27e) is an immediate consequence of the compositionality schema (2) and the specifications of the pertinent semantic operations in (26); and it brings out clearly how $*$-operations
p. 290 may operate in opaque positions, including those that are themselves in ↳ opaque environments.[20] Hence there is no need to resort to alternative compositional maneuvers along the following lines:

> $$\begin{aligned}&[\![\mathscr{P}\,(\Delta_{Norman},\ \mathscr{A}\,(\Delta_{hears},\ \mathscr{P}\,(\Delta_{syd},\ \mathscr{A}\,(\Delta_{sees}, \mathscr{P}\,(\Delta_{Emily,}\, \Delta_{\mathscr{P}lays})))))]\!]^{\,i}\\ (28)\ &= \|\mathscr{P}\|_*\Big(\ [\![\Delta_{Norman}]\!]^{\,i}, \|\mathscr{A}\|_*(\ [\![\Delta_{hears}]\!]^{\,i},\\ &\|\mathscr{P}\|_{**}\,(\ [\![\Delta_{syd}]\!]\ \ \hat{}\,, \|\mathscr{A}\|_{**}\,(\ [\![\Delta_{sees}]\!]\ \ \hat{}\,, \|\mathscr{P}\|_{***}\,(\ [\![\Delta_{Emily}]\!]\ \ \hat{}\,\hat{}\,,\ \ [\![\Delta_{\mathscr{P}lays}]\!]\ \ \hat{}\,\hat{}\,)))\,))\\ &= \|\mathscr{P}\|_*\,(n, \|\mathscr{A}\|_*\,(H_i, \|\mathscr{P}\|_{**}\,(s, \|\mathscr{A}\|_{**}\,(s, \|\mathscr{P}\|_{***}\,(\ [\![\Delta_{Emily}]\!]\ \ \hat{}\,\hat{}\,,\ \ [\![\Delta_{\mathscr{P}lays}]\!]\ \ \hat{}\ \hat{}\,)))))\end{aligned}$$

where we mark intensions by hats; double hats, triple hats etc. indicate intensions, inintensions, in-in-intensions (whatever they may be) etc.; and multiple asterisks mark suitable versions of *-operations in increasingly embedded opaque environments.

That iterated opacity as such does not motivate a hierarchy of intensions does not mean that the latter is incompatible with selective compositionality, let alone an incoherent idea. In fact, there is a straightforward way of making sense of (28), thus bringing in intensions, in-in-tensions, etc., even though they turn out to be redundant: starting from the (ordinary) intension of an expression, its indirect, in-indirect, etc., intensions may be taken to be constant functions on D_s that specify their predecessor:

> (29) *Definition*[21]
>
> If Δ is any expression, then:
>
> - $[\![\Delta]\!]^{\wedge^1} \overset{\text{def}}{=} [\![\Delta]\!]^{\wedge}$
> - $[\![\Delta]\!]^{\wedge^{n+1}} \overset{\text{def}}{=} \lambda i \in D_{\boldsymbol{s}}.\ [\![\Delta]\!]^{\wedge^n}$

The indirect intensions in (29) are not new in that they already appear in the standard type hierarchy as objects of types (*sa*); (*s*(*sa*)); (*s*(*s*(*sa*))); etc.; but they are not normally associated with expressions Δ in the above way. Two features of the construction in (29) are worth noticing:

- Increasingly indirect intensions, are increasingly complex (in set-theoretic terms) and thus distinct from all their predecessors.

- All indirect intensions are uniquely determined by the first one.

The first feature distinguishes (29) from collapsing all higher intensions into ordinary ones—which would certainly go against the Fregean spirit. Rather, the indirect intensions defined in (29) form a true hierarchy. The second feature settles potential issues of indeterminacy and learnability that are sometimes brought up in connection with Frege's hierarchy of senses:[22] to every ordinary intension there is a uniquely ↳ p. 291 identifiable and systematically derivable indirect intension of any particular order; in Parsons's (1981) terms, the hierarchy in (29) is *rigid* (as opposed to *libertine*).

Given the hierarchy in (29), it should not come as a surprise that the (doublestarred) embedded semantic operations in (28) can be defined so as to arrive at the same interpretation as in (27):

> (30)
>
> a. $\|\mathscr{P}\|_{**}(x, P) = \lambda j \in D_{\boldsymbol{s}}.\, \|\mathscr{P}\|_{*}(x_j, P_j) \quad x \in D_{\boldsymbol{se}}, P \in D_{\boldsymbol{s(et)}}$
>
> b. $\|\mathscr{A}\|_{**}(A, \varphi) = \lambda j \in D_{\boldsymbol{s}}.\, \|\mathscr{A}\|_{*}(A_j, \varphi_j) \quad A \in D_{\boldsymbol{s((st)(et))}}, \varphi \in D_{\boldsymbol{s(st)}}$
>
> c. $\|\mathscr{P}\|_{***}(\boldsymbol{x}, \boldsymbol{P}) = \lambda j \in D_{\boldsymbol{s}}.\, \|\mathscr{P}\|_{*}(\boldsymbol{x}_j, \boldsymbol{P}_j) \quad \mathrm{x} \in D_{\boldsymbol{s(se)}}, \boldsymbol{P} \in D_{\boldsymbol{s}(\mathrm{s}(\boldsymbol{et}))}$

Incidentally, the indirect combination $\|\mathscr{P}\|_{**}$ turns out to be the ordinary compositional interpretation $\|\mathscr{P}\|$ of predication; this is as it should be: the operation combines the subject and predicate intensions into a corresponding proposition that serves as the compositional value (in the opaque position) of $\|\mathscr{A}\|_{*}$. On the other hand, $\|\mathscr{A}\|_{**} \neq \|\mathscr{A}\|$, although both operations output predicate intensions; but they differ in their second argument place, which is occupied by a propositional concept in the former case and a plain proposition in the latter.

The equations in (30) trade on the fact that the arguments of the embedded operations are uniquely determined by the corresponding ordinary intensions: even though an indirect intension will be more complex than the ordinary one, the latter can be identified by applying the former to any index as many times as necessary; and so the values of the operations in (30) will eventually involve the argument of the original semantic operations. The reader is invited to verify that they do indeed do their job and have (29) come down to (27-d). A more systematic account of the pertinent operations on indirect intensions can be found in the *IL*-formalization in Appendix B, where the reformulation (28) of (27) is generalized to arbitrarily complex embeddings. So the very idea of climbing up the hierarchy of intensions with every opaque embedding turns out to be compatible with the standard account; yet it does not seem to add anything but redundancy and complications.

10.5 From Frege to Bäuerle

Though the hierarchy (29) only leads to a baroque reformulation of ordinary (uniform selectively extensional) compositionality, a slightly more flexible construction of indirect intensions turns out to be much more powerful, lending itself to a natural solution of a certain kind of scope paradox[23]. As observed in Bäuerle (1983), a sentence like (31) has a reading according to which Syd sees that each of a certain group of persons, which—(possibly) unbeknownst to Syd—happens to coincide with the band, is having a beer:

> (31) Syd sees that every band member is drinking.

p. 292 (32) expresses the relevant reading of (31) within the above setting, extended by (standard) operations $\mathscr{D}$ and $\mathscr{S}$ for dealing with quantificational subjects, as defined in (33):

> (32) $\|\mathscr{P}\|_{*}(\mathrm{s}, \|\mathscr{A}\|_{*}(S_i, \lambda j.\, \|\mathscr{S}\|_{*}(\|\mathscr{D}\|_{*}(\forall, B_i), D_j)))$
> $S_i(\mathrm{s}, \lambda j.\, \forall(B_i, D_j)) \qquad [= S_i(\mathrm{s}, \lambda j.\, \forall(B_i)(D_j)),\ \text{by notational convention}]$

> (33)
>
> a. $\|\mathscr{S}\|(Q, P)(i) = Q_i(P_i)$
>
> b. $\|\mathscr{D}\|(D, P)(i) = D_i(P_i)$

It is not obvious how to arrive at (32) in a compositional way, using standard techniques of Logical Form construction. A surface analysis (34a) yields a straightforward *de dicto* reading; and the above-mentioned construction $\mathscr{Q}$ of quantification is of no avail either, since it only allows the construals (34b) and (34c):

> (34)
>
> a. $\|\mathscr{P}\|_{*}\,(\mathrm{s}, \|\mathscr{A}\|_{*}\,(S_i, \lambda j.\, \|\mathscr{S}\|_{*}\,(\|\mathscr{D}\|_{*}(\forall, B_j), D_j)))$
> $= \quad S_i(\mathrm{s}, \lambda j.\, \forall(B_j, D_j))$
>
> b. $\|\mathscr{P}\|_{*}(\mathrm{s}, \|\mathscr{A}\|_{*}(S_i, \lambda j.\, \|\mathscr{Q}\|_{*}(\|\mathscr{D}\|_{*}(\forall, B_j), \lambda \mathrm{x}.\, \|\mathscr{P}\|_{*}(\mathrm{x}, D_j))))$
> $= \quad S_i(\mathrm{s}, \lambda j.\, \forall(B_j, \lambda \mathrm{x}.\, D_j(\mathrm{x})))$
>
> c. $\|\mathscr{Q}\|_{*}(\|\mathscr{D}\|_{*}(\forall, B_i), (\lambda \mathrm{x}.\, \|\mathscr{P}\|_{*}(\mathrm{s}, \|\mathscr{A}\|_{*}(S_i, \lambda j.\, \|\mathscr{P}\|_{*}(\mathrm{x}, D_j)))))$
> $= \quad \forall(B_i, \lambda \mathrm{x}.\, S_i(\mathrm{s}, \lambda j.\, D_j(\mathrm{x})))$

(34a) and (34b) are equivalent,[24] by (meta-linguistic) η-conversion; and in (34c) the scope of the embedded subject is too wide, even though the extension of the restrictor is as desired.[25] And surely, the baroque approach from Section 10.4 has nothing to offer here: it coincides with the selectively extensional analysis (34a), given that there is no nested opacity in (31):

> $\|P\|_{*}(\ \llbracket \Delta_{Syd} \rrbracket^{\ i}, \|A\|_{*}(\ \llbracket \Delta_{sees} \rrbracket^{\ i}$
> $(\|S\|_{**}(\big\|D\big\|_{**}(\ \llbracket \Delta_{every} \rrbracket^{\ \wedge},\ \llbracket \Delta_{band\,member} \rrbracket^{\ \wedge}),\ \llbracket \Delta_{is\,drinking} \rrbracket^{\ \wedge}))))$
> (35) $\big\|P\big\|_{*}(\ \llbracket \Delta_{Syd} \rrbracket^{\ i}, \big\|A\big\|_{*}(\ \llbracket \Delta_{sees} \rrbracket^{\ i}$
> $(\big\|S\big\|(\big\|D\big\|(\ \llbracket \Delta_{every} \rrbracket^{\ \wedge},\ \llbracket \Delta_{band\,member} \rrbracket^{\ \wedge}),\ \llbracket \Delta_{is\,drinking} \rrbracket^{\ \wedge}))))$
> $S_i(\mathrm{s}, \lambda j.\, \forall(B_j, \lambda \mathrm{x}.\, D_j(\mathrm{x})))$

p. 293 In (35), the double-star interpretations of the quantificational constructions are meant to be spelt out in analogy to the interpretation (30a) of $||P||_{*}$, since both $\mathscr{S}$ and $\mathscr{D}$ are ↳ (uniformly) extensional, according to (33); in effect, both boil down to the intensional operations defined in (33). So, as always, the baroque strategy takes a detour to where the standard approach arrives much more directly. However, a closer look at its route reveals that it could almost have led to the desired reading (32): a slight relaxation of the strategy behind (35) paves the way toa solution of Bäuerle's puzzle.[26] To see this, it is instructive to derive the intension of (31), as would be necessary when the sentence gets embedded:

> (36) Norman hears that Syd sees that every band member is drinking.

(36) has a reading according to which the contribution of (31) is precisely (the intension of) Bäuerle's reading (32):

> (37) $$\begin{aligned} & \|P\|_*(\mathrm{n}, \|A\|_*(H_i, \lambda j.\, \|P\|_*(\mathrm{s}, \|A\|_*(S_j, \lambda k.\, \|S\|_*(\|D\|_*(\forall, B_j), D_k))))) \\ = \quad & H_i(\mathrm{n}, \lambda j.\, S_j(\mathrm{s}, \lambda k.\, \forall(B_j, D_k))) \end{aligned}$$

Again the reading cannot be obtained by standard interpretive mechanisms. In particular, a standard surface analysis ultimately yields:

> (38) $H_i(\mathrm{n}, \lambda j.\, S_j(\mathrm{s}, \lambda k.\, \forall(B_k, D_k)))$

where the restrictor is interpreted *de dicto* again. And of course, so does the baroque approach. Still, it is instructive to watch closely *how* the latter derives (38). To begin with, a tedious calculation using the above definitions and abbreviations[27] brings out:

> (39) $$\begin{aligned} & \llbracket P(\Delta_{Norman}, \mathscr{A}(\Delta_{hears}, P(\Delta_{Syd}, \mathscr{A}(\Delta_{sees}, \\ & \qquad \mathscr{S}(\mathscr{D}(\Delta_{every}, \Delta_{band\,member}), \Delta_{is\,drinking})))))\rrbracket^{i} \\ & \big\|P\big\|_*(\llbracket \Delta_{Norman}\rrbracket^{i}, \big\|\mathscr{A}\big\|_*(\llbracket \Delta_{hears}\rrbracket^{i}, \big\|P\big\|_{**}(\llbracket \Delta_{Syd}\rrbracket^{\wedge}, \big\|\mathscr{A}\big\|_{**}(\llbracket \Delta_{sees}\rrbracket^{\wedge}, \\ & \qquad \big\|\mathscr{S}\big\|_{***}(\big\|\mathscr{D}\big\|_{***}(\llbracket \Delta_{every}\rrbracket^{\wedge\wedge}, \llbracket \Delta_{band\,member}\rrbracket^{\wedge\wedge}), \llbracket \Delta_{is\,drinking}\rrbracket^{\wedge\wedge}))))) \\ & H_i(\mathrm{n}, \lambda j.\, S_j(\mathrm{s}, \|\mathscr{S}\|_{***}(\|\mathscr{D}\|_{***}([\lambda j.\, \lambda k.\, \forall], [\lambda j.\, \lambda k.\, B_k]), [\lambda j.\, \lambda k.\, D_k]))(j)) \\ & H_i(\mathrm{n}, \lambda j.\, S_j(\mathrm{s}, \lambda k.\, \forall(B_k, D_k))) \end{aligned}$$

In order to reduce the triple stars on the two (fully and uniformly extensional) operations one may proceed in analogy to (30c) above and put:

> (40)
>
> a. $\|\mathscr{S}\|_{***}(\boldsymbol{Q}, \boldsymbol{P}) = \lambda k \in D_{\boldsymbol{s}}.\, \|\mathscr{S}\|_{**}(\boldsymbol{Q}_k, \boldsymbol{P}_k)$
>
> b. $\|\mathscr{D}\|_{***}(\boldsymbol{D}, \boldsymbol{P}) = \lambda k \in D_{\boldsymbol{s}}.\, \|\mathscr{D}\|_{**}(\boldsymbol{D}_k, \boldsymbol{P}_k)$

… where $\boldsymbol{Q} \in D_{(s(s((et)t)))}$, $\boldsymbol{P} \in D_{(s(s((et)))}$, and $\boldsymbol{D} \in D_{(s(s((et)((et)t))))}$. One detail of (39) is particularly noteworthy, viz. that the arguments of the triple-star operations are all indirect intensions (of some expression 6) of the form

> (41) $[\lambda j.\lambda k.\alpha]$

p. 294 where *j* does not occur in α. Given the way the ***-operations have been defined in (30c) and (40) above, the effect is that they operate on the extension of Δ at the *local index*—the index that is shifted ($\approx \lambda$-bound) by the intensional operator in whose immediate scope (the relevant occurrence of) Δ lies—in this case the intension *S* of *see*.[28] In particular, then, the indirect intension

> (42)
>
> a. $[\lambda j.\lambda k.B_k]$

ends up contributing its extension at the indices that describe the visual content of the seeing subject, Syd, thus giving rise to the *de dicto* reading. For the Bäuerle reading, however, the extension of *band member* would have to be determined according to the intended content of the embedded clause that describes the auditive content of the hearing subject, Norman, by abstraction from the outermost intensional index (corresponding to the bound variable '*j*'). However, other things being equal, the Bäuerle reading would have ensued, had the indirect intension of *band member* been

> (42)
>
> b. $[\lambda j.\lambda k.B_j]$

rather than (42a), as the reader is heartily invited to verify in detail. So the baroque approach to compositionality does offer the means to solve Bäuerle's scope paradox if the hierarchy (29) is allowed to contain alternative intensions like (42b) and the constructions employed in (31) are permitted to make use of them. In fact, it would seem that the paradox is perfectly general in that any quantificational restrictor in an opaque environment may contribute its extension relative to any index that is abstracted from by any intensional operator in whose scope it lies.[29] Thus sentences like the following appear to be (at least) four ways ambiguous:

> (43) Roger thinks that Norman hears that Syd sees that every band member is drinking.

More generally, if $\mathscr{D}(\Delta_\delta)(\Delta_\rho)$ isa nominal quantifier (consisting of a determiner Δ_δ and a restrictor Δ_ρ) occurring within a sentence, or any other expression, that has a reading of the form (44a), then the sentence also has the reading (44b), where a higher operator determines the indices i_m at which the extension of the restrictor is evaluated:

> (44)
>
> a. $\ldots[\lambda i_m.\ldots\lambda i_n.\ldots\big\|\mathscr{D}\big\|_*(\llbracket\Delta_\delta\rrbracket^{i_n}, \llbracket\Delta_\rho\rrbracket^{i_n})\ldots]$
>
> b. $\ldots[\lambda i_m.\ldots\lambda i_n.\ldots\big\|\mathscr{D}\big\|_*(\llbracket\Delta_\delta\rrbracket^{i_m}, \llbracket\Delta_\rho\rrbracket^{i_n})\ldots]$

p. 295 Moreover, in order to cover the original example (31), the variable 'i_m' in (44) would have to be allowed to coincide with the free index variable that stands for the 'actual' ↳ point of reference at which the extension of the hosting expression is determined, so that (44b) would represent the intension of the entire expression (and the initial omission marks stand for the empty string). On the baroque approach, the above constellation (44a) comes out as:

> (45) $\ldots[\ldots\big\|\mathscr{D}\big\|_*(\llbracket\Delta_\delta\rrbracket^{\wedge\cdots}, \llbracket\Delta_\rho\rrbracket^{\wedge\cdots})\ldots]^{\wedge\ldots}$

where the indirect extension $\llbracket\Delta_\rho\rrbracket^{\wedge\cdots}$ of the restrictor is of the form (46a)—which should be twisted around to derive the additional reading (46b):

> (46)
>
> a. $\llbracket\Delta_\rho\rrbracket^{\wedge\cdots} = \lambda i_1.\ldots\lambda i_m.\ldots\lambda i_n.\ \llbracket\Delta_\rho\rrbracket^{i_n}$
>
> b. $\llbracket\Delta_\rho\rrbracket^{\wedge\cdots}_{m} = \lambda i_1.\ldots\lambda i_m.\ldots\lambda i_n.\ \llbracket\Delta_\rho\rrbracket^{i_m}$

So the addition of *twisted senses* like $\llbracket\Delta_\rho\rrbracket^{\wedge\cdots}_{m}$ may improve the baroque framework and make it fit for Bäuerle's challenge.

It is obvious that twisted senses like (46b) can be systematically defined in terms of corresponding indirect intensions like (46a); the construction will be given in detail in Appendix B. However, to complete the account of the scope paradox, their precise rôle still needs to be determined. In principle, there are two ways in which twisted senses might enter meaning composition: they could be introduced by lexical processes, or they could be called up by (some) semantic operations. The second option seems preferable because, on the one hand, indefinites seem to be subject to the particular shift from (46a) to (46b), but not if they are used predicatively.[30] Thus, for example, (47) does not seem to have a reading according to which Syd said something about a certain group of people who, possibly unbeknownst to Syd, happen to be the members of the band.

> (47) Syd said that every brother of Emily's <u>is a band member</u>.

It is hard to see how a lexically based account of twisted senses could avoid that they are employed in (47) when the underlined predicate is formed out of the twisted reading of the indefinite *a band member*. On the other hand, it is quite conceivable that certain *flexible* constructions be endowed with alternative readings that access twisted senses when need be. In particular, this flexibility may be attributed to the formation $\mathscr{D}$ of quantifiers but not the constructions underlying the formation of predicate nominatives; alternatively, flexibility may only come in in quantificational constructions like $\mathscr{Q}$ and $\mathscr{S}$. Appendix B contains an implementation of the first option to a highly restricted fragment of English in terms of indirect interpretation; the reader is invited to explore the second one.[31] On either approach, there are at least three ways in which flexibility may be accounted for:

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- ↳ *Disambiguation by construction*: Flexible operations could be multiply ambiguous according to which twisted sense they are applied to.
- *Disambiguation by parameterisation*: There could be a parameter in the object language that regulates which twisted sense a flexible operation is applied to in a given expression.
- *Underspecification*: The interpretation collects all readings deriving from applying flexible operations to twisted senses of their (pertinent) arguments in one set.

The implementation in the appendix follows the third interpretational strategy; this choice has been made mainly for expository reasons. It may be worth emphasizing that, unlike other approaches to Bäuerle's scope paradox (and related phenomena), the interpretation employs Montague's (1970) intensional type logic *IL* and thus does without any variables of type ***s***.[32] Whether it is a viable alternative to them ultimately depends on whether the highly constrained environments in which such constellations occur, can be characterized exclusively in terms of the scopal positions encoded by the (twisted) indirect intensions.

It thus turns out that, whatever Frege's own motivation behind the baroque approach to compositionality may have been, it does have its merits when equipped with the right kind of indirect intensions. However, like the rigid intensions in (29), their twisted versions are a far cry from the Fregean hierarchy of senses, even though they are libertine in that two expressions, in fact even two occurrences of the same expression, may be co-intensional without agreeing in all their twisted senses. Still, any two expressions with the same *ordinary* intensions will have the same set of twisted senses. It is this feature that immunizes the hierarchy of twisted senses against non-learnability objections; but it also makes it a bad candidate for an accurate reconstruction of Frege's hierarchy, even taking obvious frameworkdependent limitations into account.

10.6 Conclusion

Two distinctive features of Frege's approach to compositionality have been reconstructed in terms of the theory of extension and intension: (*i*) its bias in favour of extensional operations and (*ii*) its resort to indirect senses in the face of iterated opacity. While (*i*) has been preserved in current formal semantics, it proves to be stronger than a straightforward p. 297 extensionality requirement in terms of Logical Space, the difference turning on a subtle distinction between extensions *at* particular ↳ points and extensions *per se*. (*ii*) has traditionally been dismissed as redundant, and was shown to lead to a mere 'baroque' reformulation of ordinary compositionality. Nevertheless, whatever Frege's motive, the very idea of having opaque denotations keep track of the depth of their embedding gives rise to a fresh view at certain scope paradoxes that had been argued to lie outside the reach of a binary distinction between extension and intension.

Appendix A: (39) Under the Microscope

> (48)

$$
\begin{array}{lll}
 & \llbracket \mathscr{P}(\Delta_{Norman}, \mathscr{A}(\Delta_{hears}, & \\
 & \mathscr{P}(\Delta_{Syd}, \mathscr{A}(\Delta_{sees}, \mathscr{S}(\mathscr{D}(\Delta_{every}, \Delta_{band\,member}), \Delta_{is\,drinking})))))\rrbracket^{\ i} & \\
= & \|\mathscr{P}\|_{*}(\ \llbracket\Delta_{Norman}\rrbracket^{\ i}, \|\mathscr{A}\|_{*}(\ \llbracket\Delta_{hears}\rrbracket^{\ i}, & = (39) \\
 & \llbracket\|P\|_{**}(\ \llbracket\Delta_{Syd}\rrbracket^{\ \wedge}, \|\mathscr{A}\|_{**}(\ \llbracket\Delta_{sees}\rrbracket^{\ \wedge}, & \\
 & \|\mathscr{S}\|_{***}(\|\mathscr{D}\|_{***}(\ \llbracket\Delta_{every}\rrbracket^{\ \wedge\wedge},\ \llbracket\Delta_{band\,member}\rrbracket^{\ \wedge\wedge}),\ \llbracket\Delta_{is\,drinking}\rrbracket^{\ \wedge\wedge}))))) & \\
= & H_i(\mathrm{n}, \|\mathscr{P}\|_{**}([\lambda j.\ \mathrm{s}], \|\mathscr{A}\|_{**}([\lambda j.\ S_j], & (26)\&\ \text{abbreviations} \\
 & \|\mathscr{S}\|_{***}(\|\mathscr{D}\|_{***}([\lambda j.\ \lambda k.\ \forall], [\lambda j.\ \lambda k.\ \mathbf{B}_k]), [\lambda j.\ \lambda k.\ D_k])))) & \\
= & H_i(\mathrm{n}, \lambda j.\ \|\mathscr{A}\|_{***}([\lambda j.\ S_j], & (3) : \|\mathscr{P}\|_{**} = \|\mathscr{P}\|! \\
 & \|\mathscr{S}\|_{***}(\|\mathscr{D}\|_{***}([\lambda j.\ \lambda k.\ \forall], [\lambda j.\ \lambda k.\ \mathbf{B}_k]), [\lambda j.\ \lambda k.\ D_k]))\ (j)([\lambda j.\ \mathrm{s}](j))) & \\
= & H_i(\mathrm{n}, \lambda j.\ [\lambda k.\ [\lambda j.\ S_j](k) & (30\mathrm{b}) \\
 & (\|\mathscr{S}\|_{***}(\|\mathscr{D}\|_{***}([\lambda j.\ \lambda k.\ \forall], [\lambda j.\ \lambda k.\ \mathbf{B}_k]), [\lambda j.\ \lambda k.\ D_k]))\ (k)](j)([\lambda j.\ \mathrm{s}](j))) & \\
= & H_i(\mathrm{n}, \lambda j.\ [\lambda k.\ S_k & \beta - \text{conversion} \\
 & (\|\mathscr{S}\|_{**}(\|\mathscr{D}\|_{***}([\lambda j.\ \lambda k.\ \forall], [\lambda j.\ \lambda k.\ B_k]), [\lambda j.\ \lambda k.\ D_k]))\ (k)](j)([\lambda j.\ \mathrm{s}](j))) & \\
= & H_i(\mathrm{n}, \lambda j.\ S_j(\|\mathscr{S}\|_{***}(\|\mathscr{D}\|_{***}([\lambda j.\ \lambda k.\ \forall], [\lambda j.\ \lambda k.\ B_k]), [\lambda j.\ \lambda k.\ D_k]))\ (j)(\mathrm{s})) & \\
 & \qquad\qquad\qquad\qquad\qquad\qquad \beta - \text{conversion} & \\
= & H_i(\mathrm{n}, \lambda j.\ S_j(\mathrm{s},\ \|\mathscr{S}\|_{***}(\|\mathscr{D}\|_{***}([\lambda j.\ \lambda k.\ \forall], [\lambda j.\ \lambda k.\ B_k]), [\lambda j.\ \lambda k.\ D_k]))\ (j)) & \\
 & \qquad\qquad\qquad\qquad\qquad\qquad \beta - \text{conversion} &
\end{array}
$$

The contribution of (31) can be determined separately:

> (49)

$$
\begin{array}{ll}
\|\mathscr{S}\|_{***}(\|\mathscr{D}\|_{***}([\lambda j. \lambda k.\ \forall], [\lambda j. \lambda k.\ B_k]), [\lambda j. \lambda k.\ D_k]) & \\
\lambda k.\ \|\mathscr{S}\|_{**}(\|\mathscr{D}\|_{***}([\lambda j. \lambda k.\ \forall], [\lambda j. \lambda k.\ B_k])(k), [\lambda j. \lambda k.\ D_k]\ (k)) & (40\mathrm{a}) \\
\lambda k.\ \|\mathscr{S}\|_{**}(\|\mathscr{D}\|_{**}([\lambda j. \lambda k.\ \forall], [\lambda j. \lambda k.\ B_k])(k)), [\lambda j. \lambda k.\ D_k]\ (k)) & (40\mathrm{b})\&\beta - \text{conversion} \\
\lambda k.\ \|\mathscr{S}\|_{**}(\|\mathscr{D}\|_{**}(\lambda k.\ \forall, \lambda k.\ B_k), \lambda k.\ D_k) & \beta - \text{conversion} \\
\lambda k. \lambda j. [\lambda j. \forall(B_j)](j)([\lambda j. D_j](j)) & (33\mathrm{a}) :\ \llbracket\mathscr{S}\rrbracket_{\ **} =\ \llbracket\mathscr{S}\rrbracket\ ! \\
\lambda k. \lambda j. \forall(B_j, D_j) & \beta - \text{conversion}\ \&\ \text{notation}
\end{array}
$$

Inserting the result of (49) into (48) yields:

$$H_i(\mathrm{n}, \lambda j. S_j(s, [\lambda k. \lambda j. \forall(B_j, D_j)](j))),$$

which β-reduces to:

> $H_i(\mathrm{n}, \lambda j.\, S_j(s, \lambda j.\, \forall(B_j, D_j)))$,

as expected and desired.

p. 298 Appendix B: *IL*-implementation

(a) Syntax of fragment

The fragment covers the key examples in the main text. The lexicon contains the following sets of expressions:

- *Names*: Δ_{Emily}, Δ_{Norman}, Δ_{Syd}, …
- *Predicates*: Δ_{plays}, $\Delta_{is\ drinking}$, …
- *Attitude Verbs*: Δ_{sees}, Δ_{hears}, …
- *Nouns*: $\Delta_{band\ member}$, …
- *Determiners*: Δ_{every}, Δ_{a}, …

The syntax comprises the constructions discussed above and contains the following rules:

R1 If Δ_{NN} is a *Name* and Δ_P is a *Predicate*, then $\mathscr{P}(\Delta_{NN}, \Delta_P)$ is a *Sentence*.

R2 If Δ_A is an *Attitude Verb* and Δ_S is a *Sentence*, then $\mathscr{A}(\Delta_A, \Delta_S)$ is a *Predicate*.

R3 If Δ_D is a *Determiner* and Δ_N is a *Noun*, then $\mathscr{D}(\Delta_D, \Delta_N)$ is a *Quantifier*.

R4 If Δ_Q is a *Quantifier* and Δ_P is a *Predicate*, then $\mathscr{Q}(\Delta_Q, \Delta_P)$ is a *Sentence*.

(b) *IL*: definitions and notation

Basic concepts

The interpretation of the fragment will proceed indirectly, by way of a compositional translation into Montague's (1970) language *IL* of intensional type logic. The language is based on infinite sets Var_a of variables of any type a and unspecified sets Con_a of (non-logical) constants of type a, and consists of a set IL_a of terms of (any) type a:

- $Var_a \subseteq IL_a$.
- $Con_a \subseteq IL_a$.
- If $\alpha \in IL_{ab}$ and $\beta \in IL_a$, then $\alpha(\beta) \in IL_b$.
- If $\mathbf{x} \in IL_a$ and $\alpha \in IL_b$, then $(\lambda\mathbf{x}.\ \alpha) \in IL_{ab}$.
- If $\alpha \in IL_a$ and $\beta \in IL_a$, then $(\alpha = \beta) \in IL_{\mathbf{t}}$.
- If $\alpha \in IL_{\mathbf{s}a}$, then $[{}^{\vee}\alpha] \in IL_a$.
- If $\alpha \in IL_a$, then $[{}^{\wedge}\alpha] \in IL_{\mathbf{s}a}$.

Following Montague (1970), logical constants and operators (like ∨, ∧, ∃ and ∀) may be taken as abbreviations. *IL*-terms receive their denotations relative to models $\mathscr{M} = (D_e, D_s, \mathbf{F})$, indices $i \in D_s$, and $\mathscr{M}$-assignments g:

- $[\![\boldsymbol{x}]\!]^{\mathscr{M},i,g} = g(\boldsymbol{x})$ if $\boldsymbol{x} \in Var_a$.
- $[\![\mathbf{c}]\!]^{\mathscr{M},i,g} = \mathbf{F}(\mathbf{c})(i)$ if $\mathbf{c} \in Con_a$.
- $[\![\alpha(\beta)]\!]^{\mathscr{M},i,g} = [\![\alpha]\!]^{\mathscr{M},i,g}([\![\beta]\!]^{\mathscr{M},i,g})$.
- $[\![(\lambda\boldsymbol{x}^a.\, \alpha)]\!]^{\mathscr{M},i,g} = \left\{(u, [\![\alpha]\!]^{\mathscr{M},i,g[{}^{\mathrm{x}}/_{u}]}) \middle| u \in D_a\right\}$.
- $[\![(\alpha = \beta)]\!]^{\mathscr{M},i,g} = \left\{\emptyset \middle| [\![\alpha]\!]^{\mathscr{M},i,g} = [\![\beta]\!]^{\mathscr{M},i,g}\right\}$.[33]
- p. 299 ↳ $[\![[{}^{\vee}\alpha]]\!]^{\mathscr{M},i,g} = [\![\alpha]\!]^{\mathscr{M},i,g}(i)$.
- $[\![[{}^{\wedge}\alpha]]\!]^{\mathscr{M},i,g} = \left\{(j, [\![\alpha]\!]^{\mathscr{M},j,g}) \middle| j \in D_s\right\}$.

Two *IL*-terms α and β of the same type are *logically equivalent* iff $[\![\alpha]\!]^{\mathscr{M},i,g} = [\![\beta]\!]^{\mathscr{M},i,g}$, for all models $\mathscr{M}$, for all modles $\mathscr{M}$, indices i and assignments g; notation: $\alpha \equiv \beta$.

Iteration of IL-operators

The indirect intensions in the hierarchy (29) are of the types of the form $(\mathbf{s}^n a)$:

- $(\mathbf{s}^0 a) = a$
- $(\mathbf{s}^{n+1} a) = (\mathbf{s}(\mathbf{s}^n a))$

For each *IL*-term α the term $[{}^{\wedge^n}\alpha]$ denotes its n^{th} indirect intension:

- $[^{\wedge^0}] = \alpha$
- $[^{\wedge^{n+1}}\alpha] = [^{\wedge}[^{\wedge^n}\alpha]] \qquad (= [^{\wedge^n}[^{\wedge}\alpha]])$

The indirect interpretation algorithms will also make use of iterated index application $[^{\vee^n}\alpha]$:

- $[^{\vee^0}] = \alpha$
- $[^{\vee^{n+1}}\alpha] = [^{\vee}[^{\vee^n}\alpha]] \qquad (= [^{\vee^n}[^{\vee}\alpha]])$

For each *IL*-term α, any $n \geq 0$ and $m \leq n$, the term $[^{\wedge^n_m}\alpha]$ designates the mth twisted version of α's nth indirect intension:

- $[^{\wedge^n_m}\alpha] = [^{\wedge^m}(\lambda \mathrm{X}.\ [^{\wedge^{n-m}}\mathrm{X}])(\alpha)] \qquad (0 \leq m \leq n)$

Functional application is defined recursively on the hierarchy of indirect intensions and twisted senses:

- $\mathbf{A}^0_{ab} = (\boldsymbol{\lambda f}.\ \boldsymbol{\lambda x}.\ \boldsymbol{f}(\boldsymbol{x})) \qquad \boldsymbol{f} \in Var_{ab}, \boldsymbol{x} \in Var_a$
- $\mathbf{A}^{n+1}_{ab} = (\boldsymbol{\lambda f}.\boldsymbol{\lambda x}.\ [^{\wedge}\mathbf{A}^n_{ab}([^{\vee}\boldsymbol{f}])([^{\vee}\boldsymbol{x}])]) \qquad \boldsymbol{f} \in Var_{(\mathbf{s}^{n+1}(ab))}, \boldsymbol{x} \in Var_{(\boldsymbol{s}^{n+1}a)}$

(c) Indirect interpretation

Standard translation

For each expression Δ from the fragment defined in (a) the *IL*-term |Δ| denotes its extension:

- $\{|\Delta_{Emily}|, |\Delta_{Norman}|, |\Delta_{Syd}|, \ldots\} = \{\mathbf{e}, \mathbf{n}, \mathbf{s}, \ldots\} \subseteq Con_{\boldsymbol{e}}$
- $\{|\Delta_{plays}|, |\Delta_{is\ drinking}|, \ldots\} = \{\mathbf{P}, \mathbf{D}, \ldots\} \subseteq Con_{\boldsymbol{et}}$
- $\{|\Delta_{sees}|, |\Delta_{hears}|, \ldots\} = \{\mathbf{S}, \mathbf{H}, \ldots\} \subseteq Con_{(\boldsymbol{st})(\boldsymbol{et})}$
- $\{|\Delta_{band\ member}|, \ldots\} = \{\mathbf{B}, \ldots\} \subseteq Con_{\boldsymbol{et}}$
- $\left|\Delta_{every}\right| = [\boldsymbol{\lambda P^{et}}.\ \boldsymbol{\lambda Q^{et}}.\ (\forall \boldsymbol{x}^{\boldsymbol{e}})[\boldsymbol{P}(\boldsymbol{x}) \rightarrow \boldsymbol{Q}(\boldsymbol{x})] \qquad \left[\overset{\text{def}}{=} \mathbf{ALL}\right]$
- $|\Delta_a| = [\boldsymbol{\lambda P^{et}}.\boldsymbol{\lambda Q^{et}}.\ (\exists \boldsymbol{x}^{\boldsymbol{e}})[\boldsymbol{P}(\boldsymbol{x}) \wedge \boldsymbol{Q}(\boldsymbol{x})]]$

S1 $|\mathscr{P}(\Delta_{NN}, \Delta_P)| = |\Delta_P|(|\Delta_{NN}|)$

S2 $|\mathscr{A}(\Delta_A, \Delta_S)| = |\Delta_A|([^{\wedge}|\Delta_S|])$

S3 $||\mathscr{D}(\Delta_D, \Delta_N)| = |\Delta_D|(|\Delta_N|)$

S4 $|\mathscr{Q}(\Delta_Q, \Delta_P)| = |\Delta_Q|(|\Delta_P|)$

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Baroque translation

For each expression Δ from the fragment defined in (a) the *IL*-term $|\Delta|^n$ denotes its nth indirect intension, which coincides with its extension if $n = 0$. In opaque positions the translation increases the level of indirectness:

- $\left|\Delta\right|^n = {}^{\wedge^n}\left|\Delta\right|$ if Δ is lexical

B1 $|\mathscr{P}(\Delta_{NN}, \Delta_P)|^n = \mathbf{A}^{\mathrm{n}}_{\boldsymbol{et}}(|\Delta_P|^n)(|\Delta_{NN})|^n)$

B2 $\left|\mathscr{A}(\Delta_A, \Delta_S)\right|^n = \mathbf{A}^n_{(\boldsymbol{st})(\boldsymbol{et})}(\left|\Delta_A\right|^n)(\left|\Delta_S)\right|^{n+1})$

B3 $\left|\mathscr{D}(\Delta_D, \Delta_N)\right|^n = \mathbf{A}^n_{(\boldsymbol{et})((\boldsymbol{et})\boldsymbol{t})}(\left|\Delta_D\right|^n)(\left|\Delta_N)\right|^n)$

B4 $\left|\mathscr{Q}(\Delta_Q, \Delta_P)\right|^n = \mathbf{A}^n_{(\boldsymbol{et})\boldsymbol{t}}(\left|\Delta_Q\right|^n)(\left|\Delta_P)\right|^n)$

Underspecified translation

For each expression Δ from the fragment defined in (a) the set $|\Delta|^n_\sim$ of *IL*-terms contains all possible choices of nth indirect senses in flexible argument positions; in inflexible positions the semantic operations are distributed, flexible positions bring in twisted senses. The technique is the same as in Rooth's (1985) alternative semantics of focus:

- $|\Delta|^n_\sim = \{|\Delta|^n\}$ if Δ is lexical

U1 $|\mathscr{P}(\Delta_{NN}, \Delta_P)|^n_\sim = \{\mathbf{A}^{\mathrm{n}}_{\boldsymbol{et}}(\alpha)(\beta) \mid \alpha \in |\Delta_P|^n_\sim,\ \beta \in |\Delta_N N|^n_\sim\}$

U2 $\left|\mathscr{A}(\Delta_A, \Delta_S)\right|^n_\sim = \left\{\mathbf{A}^n_{(\boldsymbol{st})(\boldsymbol{et})}(\alpha)(\beta) \,\middle|\, \alpha \in \left|\Delta_A\right|^n_\sim,\ \beta \in \left|\Delta_S\right|^{n+1}_\sim\right\}$

U3
$$|\mathscr{D}(\Delta_D, \Delta_N)|^n_\sim = \left\{\mathbf{A}^n_{(\boldsymbol{et})((\boldsymbol{et})\boldsymbol{t})}(\mathrm{\alpha})(^{\wedge^n_m\vee^n}\beta) \,\middle|\, m \leq n,\ \alpha \in \left|\Delta_D\right|^n_\sim,\ \beta \in \left|\Delta_N\right|^n_\sim\right\}$$

U4 $\left|\mathscr{Q}(\Delta_Q, \Delta_P)\right|^n_\sim = \left\{\mathbf{A}^n_{(\boldsymbol{et})\boldsymbol{t}}(\alpha)(\beta) \,\middle|\, \alpha \in \left|\Delta_Q\right|^n_\sim,\ \beta \in \left|\Delta_P\right|^n_\sim\right\}$

(d) Comparison

Principal observations

Let Δ be any expression in the above fragment. Then:

P1 $|\Delta|^0 = |\Delta|$

P2 $|\Delta| \equiv \alpha \in |\Delta|^0_\sim$, for some *IL*-term α

Auxiliary observations

For all *IL*-terms α, $n \geq m \geq 0$, *IL*-models $\mathscr{M} = (D_\mathrm{e}, D_\mathrm{s}, \mathbf{F}), i_0, \ldots, i_{n+1} \in D_\mathrm{s}$, and $\mathscr{M}$-assignments g, the following hold:

A1 $\llbracket {}^{\vee^n \wedge^n}\alpha \rrbracket^{\mathscr{M},i_0,g} = \llbracket \alpha \rrbracket^{\mathscr{M},i_0,g}$

A2 $\llbracket {}^{\wedge^n}\alpha \rrbracket^{\mathscr{M},i_0,g}(i_1)\ldots(i_n) = \llbracket \alpha \rrbracket^{\mathscr{M},i_n,g}$

A3 $\llbracket {}^{\wedge^{n+1}}\alpha \rrbracket^{\mathscr{M},i_0,g}(i_1)\ldots(i_n) = \llbracket {}^\wedge\alpha \rrbracket^{\mathscr{M},i_{n+1},g}$

A4 $\llbracket \mathbf{A}^n_{ab}({}^{\wedge n}\alpha)({}^{\wedge n}\beta) \rrbracket^{\mathscr{M},i_0,g} = \llbracket {}^{\wedge n}\alpha(\beta) \rrbracket^{\mathscr{M},i_0,g}$ where, for some types a and b, $\alpha \in IL_{(\mathrm{s}^n(ab))}$, $\beta \in IL_{(\mathrm{s}^n a)}$

p. 301 *A5* ↳ $|\Delta|^n \equiv^{\wedge n} |\Delta|$

A6 $\llbracket {}^{\wedge^n_m}\alpha \rrbracket^{\mathscr{M},i_0,g}(i_1)\ldots(i_n) = \llbracket \alpha \rrbracket^{\mathscr{M},i_m,g}$

A7 $\llbracket {}^{\wedge^n_n}\alpha \rrbracket^{\mathscr{M},i_0,g} = \llbracket {}^{\wedge n}\alpha \rrbracket^{\mathscr{M},i_0,g}$

A8 $\llbracket \mathbf{A}^n_{ab}(\alpha)(\beta) \rrbracket^{\mathscr{M},i_0,g}(i_1)\ldots(i_n)$
$= \llbracket \alpha \rrbracket^{\mathscr{M},i_0,g}(i_1)\ldots(i_n)(\llbracket \beta \rrbracket^{\mathscr{M},i_0,g}(i_1)\ldots(i_n))$ where α and β are as in *A4*

A9 ${}^{\wedge n}|\Delta| \equiv \alpha \in |\Delta|^n_\sim$, for some $IL -$ term α

p. 302 (e) Example

> (36) Norman hears that Syd sees that every band member is drinking.

- *Underlying structure:*
  $\mathscr{P}(\Delta_{Norman}, \mathscr{A}(\Delta_{hears}, \mathscr{P}(\Delta_{Syd},$
  $\mathscr{A}(\Delta_{sees}, \mathscr{Q}(\mathscr{D}(\Delta_{every}, \Delta_{band\,member}), \Delta_{is\,drinking})))))$

Standard translation

- $|\mathscr{Q}(\mathscr{D}(\Delta_{every}, \Delta_{band\,member}), \Delta_{is\,drinking})|$
  $= |\Delta_{every}|(|\Delta_{band\,member}|)(|\Delta_{is\,drinking}|)$
  $= \mathbf{ALL}(\mathbf{B})(\mathbf{D})$
  $\equiv (\forall \boldsymbol{x}^{\boldsymbol{e}})\,[\mathbf{B}(\boldsymbol{x}) \to \mathbf{D}(\boldsymbol{x})]$
- $|\mathscr{P}(\Delta_{Syd}, \mathscr{A}(\Delta_{sees}, \mathscr{Q}(\mathscr{D}(\Delta_{every}, \Delta_{band\,member}), \Delta_{is\,drinking})))|$
  $= \mathbf{S}(\mathbf{s}, [{}^\wedge|\Delta_{every}|(|\Delta_{band\,member}|)(|\Delta_{is\,drinking}|)])$
  $\equiv \mathbf{S}(\mathbf{s}, [{}^\wedge(\forall \mathrm{x}^\mathrm{e})\,[\mathbf{B}(\mathbf{S}) \to \mathbf{D}(\mathbf{x})]])$
- $|(36)|$
  $= \mathbf{H}(\mathbf{n}, [{}^\wedge|\mathscr{P}(\Delta_{Syd}, \mathscr{A}(\Delta_{sees}, \mathscr{Q}(\mathscr{D}(\Delta_{every}, \Delta_{bandmember}), \Delta_{is\,drinking})))|])$
  $\equiv \mathbf{H}(\mathbf{n}, [{}^\wedge\mathbf{S}(\mathbf{s}, [{}^\wedge(\forall \mathrm{x}^\mathrm{e})\,[\mathbf{B}(\mathrm{x}) \to \mathbf{D}(\mathrm{x})]])])$
  $\Rightarrow \llbracket |(36)| \rrbracket^{\mathscr{M},i,g} = H_i(\mathrm{n}, \lambda j.\ S_j(\mathrm{s}, \lambda k.\ B_k \subseteq D_k))$
  using notational conventions from the text

p. 303 Baroque translation

$$
\begin{aligned}
& |36|^0 \\
&= \mathbf{A}^0_{\mathbf{et}}(\mathbf{A}^0_{(\mathrm{st})(\mathrm{et})}(\mathbf{H})(\mathbf{A}^1_{\mathrm{et}}(\mathbf{A}^1_{(\mathrm{st})(\mathrm{et})}({}^\wedge\mathbf{S}) \\
&\qquad (\mathbf{A}^2_{(\mathrm{et})\mathbf{t}}(\mathbf{A}^2_{(\mathrm{et})((\mathrm{et})\mathrm{t})}({}^{\wedge\wedge}\mathbf{ALL})({}^{\wedge\wedge}\mathbf{B}))({}^{\wedge\wedge}\mathbf{D})))({}^\wedge\mathbf{s})))(\mathbf{n}) \\
&= \mathbf{A}^0(\mathbf{A}^0(\mathbf{H})(\mathbf{A}^1(\mathbf{A}^1({}^\wedge\mathbf{S})(\mathbf{A}^2(\mathbf{A}^2({}^{\wedge\wedge}\mathbf{ALL})({}^{\wedge\wedge}\mathbf{B}))({}^{\wedge\wedge}\mathbf{D})))({}^\wedge\mathbf{s})))(\mathbf{n}) \\
&\qquad [\text{omitting indices for readabilir}] \\
&\equiv \mathbf{A}^0(\mathbf{A}^0(\mathbf{H})(\mathbf{A}^1(\mathbf{A}^1({}^\wedge\mathbf{S})([{}^\wedge[{}^\wedge\mathbf{ALL}(\mathbf{B})(\mathbf{D})]]))(\mathbf{s})))(\mathbf{n}) && 2 \times (A4)\ [\mathrm{n} = 2] \\
&\equiv \mathbf{A}^0(\mathbf{A}^0(\mathbf{H})([{}^\wedge\mathbf{S}([{}^\wedge\mathbf{ALL}(\mathbf{B})(\mathbf{D})])(\mathbf{s})]))(\mathbf{n}) && 2 \times (A4)\ [\mathrm{n} = 1] \\
&\equiv \mathbf{H}([{}^\wedge\mathbf{S}([{}^\wedge\mathbf{ALL}(\mathbf{B})(\mathbf{D})])(\mathbf{s})])(\mathbf{n}) && \text{def. } \mathrm{A}^0 \\
&\equiv \mathbf{H}(\mathbf{n}, [{}^\wedge\mathbf{S}(\mathbf{s}, [{}^\wedge\mathbf{ALL}(\mathbf{B})(\mathbf{D})])]) && \text{notation} \\
&\Rightarrow \llbracket |36|^0 \rrbracket^{\mathscr{M},i,g} \\
&= \llbracket \mathbf{H}(\mathbf{n}, [{}^\wedge\mathbf{S}(\mathbf{s}, [{}^\wedge\mathbf{ALL}(\mathbf{B})(\mathbf{D})])]) \rrbracket^{\mathscr{M},i,g} \\
&= H_i(\mathrm{n}, \lambda j.\ S_j(\mathrm{s}, \lambda k.\ B_k \subseteq D_k)) \\
&= \llbracket |36| \rrbracket^{\mathscr{M},i,g}
\end{aligned}
$$

Underspecified translation

$$|(36)|^0_\sim$$
$$=\{\ \mathbf{A}^0(\mathbf{A}^0(\alpha)(\mathbf{A}^1(\mathbf{A}^1(\beta)(\mathbf{A}^2(\mathbf{A}^2(\gamma)({}^{\wedge^2_m\vee^2}\delta))(\varepsilon)))(\sigma)))(\nu)\ \Big|\ m \leq 2,$$
$$\alpha \in |\Delta_{hears}|^0_\sim,\ \beta \in |\Delta_{sees}|^1_\sim,\ \gamma \in |\Delta_{every}|^2_\sim,\ \delta \in |\Delta_{band\,member}|^2_\sim,$$
$$\varepsilon \in |\Delta_{is\,drinking}|^2_\sim,\ \sigma \in |\Delta_{Syd}|^1_\sim,\ \nu \in |\Delta_{Norman}|^0_\sim\}$$
$$= \Big\{\mathbf{A}^0(\mathbf{A}^0(\mathbf{H})(\mathbf{A}^1(\mathbf{A}^1({}^\wedge\mathbf{S})(\mathbf{A}^2(\mathbf{A}^2({}^{\wedge^2}\mathbf{ALL})({}^{\wedge^2_0\vee^2\wedge^2}\mathbf{B}))({}^{\wedge^2}\mathbf{D})))({}^\wedge\mathbf{s})))(\mathbf{n}),$$
$$\mathbf{A}^0(\mathbf{A}^0(\mathbf{H})(\mathbf{A}^1(\mathbf{A}^1({}^\wedge\mathbf{S})(\mathbf{A}^2(\mathbf{A}^2({}^{\wedge^2}\mathbf{ALL})({}^{\wedge^2_1\vee^2\wedge^2}\mathbf{B}))({}^{\wedge^2}\mathbf{D})))({}^\wedge\mathbf{s})))(\mathbf{n}),$$
$$\mathbf{A}^0(\mathbf{A}^0(\mathbf{H})(\mathbf{A}^1(\mathbf{A}^1({}^\wedge\mathbf{S})(\mathbf{A}^2(\mathbf{A}^2({}^{\wedge^2}\mathbf{ALL})({}^{\wedge^2_2\vee^2\wedge^2}\mathbf{B}))({}^{\wedge^2}\mathbf{D})))({}^\wedge\mathbf{s})))(\mathbf{n})\Big\}$$
$$= \{|(36)|^0_0, |(36)|^0_1, |(36)|^0_2\}$$

$$\llbracket|36|^0_0\rrbracket^{\mathscr{M},i,g}$$
$$\overset{A8}{=}\ \llbracket\mathbf{H}\rrbracket^{\mathscr{M},i,g}(\ \llbracket\mathbf{n}\rrbracket^{\mathscr{M},i,g},$$
$$\llbracket\mathbf{A}^1(\mathbf{A}^1({}^\wedge\mathbf{S})(\mathbf{A}^2(\mathbf{A}^2({}^{\wedge\wedge}\mathbf{ALL})({}^{\wedge^2_0\vee^2\wedge\wedge}\mathbf{B}))({}^{\wedge\wedge}\mathbf{D})))({}^\wedge\mathbf{s})))\rrbracket^{\mathscr{M},i,g})$$
$$\overset{\eta\,\text{conv.}}{=}\ \llbracket\mathbf{H}\rrbracket^{\mathscr{M},i,g}(\,||\,\mathbf{n}\,||^{\mathscr{M},i,g},$$
$$\lambda j.\ \llbracket\mathbf{A}^1(\mathbf{A}^1({}^\wedge\mathbf{S})(\mathbf{A}^2(\mathbf{A}^2({}^{\wedge\wedge}\mathbf{ALL})^{\wedge^2_0\vee^2\wedge\wedge}\mathbf{B}))({}^{\wedge\wedge}\mathbf{D})))({}^\wedge\mathbf{s}))\rrbracket^{\mathscr{M},i,g}(j))$$
$$\overset{A8}{=}\ \llbracket\mathbf{H}\rrbracket^{\mathscr{M},i,g}(\ \llbracket\mathbf{n}\rrbracket^{\mathscr{M},i,g},$$
$$\lambda j.\ \llbracket\mathbf{A}^1({}^\wedge\mathbf{S})(\mathbf{A}^2(\mathbf{A}^2({}^{\wedge\wedge}\mathbf{ALL})^{\wedge^2_0\vee^2\wedge\wedge}\mathbf{B}))(\wedge\wedge\mathbf{D})))\rrbracket^{\mathscr{M},i,g}(j)(\ \llbracket{}^\wedge\mathbf{s}\rrbracket^{\mathscr{M},i,g}(j)))$$
$$\overset{IL}{=}\ \llbracket\mathbf{H}\rrbracket^{\mathscr{M},i,g}(\ \llbracket\mathbf{n}\rrbracket^{\mathscr{M},i,g},$$
$$\lambda j.\ \llbracket\mathbf{A}^1({}^\wedge\mathbf{S})(\mathbf{A}^2(\mathbf{A}^2({}^{\wedge\wedge}\mathbf{ALL})^{\wedge^2_0\vee^2\wedge\wedge}\mathbf{B}))(\wedge\wedge\mathbf{D})))\rrbracket^{\mathscr{M},i,g}(j)(\ \llbracket\mathbf{s}\rrbracket^{\mathscr{M},j,g}))$$
$$\overset{A8}{=}\ \llbracket\mathbf{H}\rrbracket^{\mathscr{M},i,g}(\ \llbracket\mathbf{n}\rrbracket^{\mathscr{M},i,g},$$
$$\lambda j.\ \llbracket{}^\wedge\mathbf{S}\rrbracket^{\mathscr{M},i,g}(j)(\ \llbracket\mathbf{A}^2(\mathbf{A}^2({}^{\wedge\wedge}\mathbf{ALL})({}^{\wedge^2_0\vee^2\wedge\wedge}\mathbf{B}))(\wedge\wedge\mathbf{D})\rrbracket^{\mathscr{M},i,g}(j)(\ \llbracket\mathbf{s}\rrbracket^{\mathscr{M},i,g}))$$
$$\overset{IL}{=}\mathbf{H}^{\mathscr{M},i,g}(\mathbf{n}^{\mathscr{M},i,g},$$
$$\lambda j.\ \llbracket\mathbf{S}\rrbracket^{\mathscr{M},j,g}(\ \llbracket\mathbf{A}^2(\mathbf{A}^2({}^{\wedge\wedge}\mathbf{ALL})({}^{\wedge^2_0\vee^2\wedge\wedge}\mathbf{B}))(\wedge\wedge\mathbf{D})\rrbracket^{\mathscr{M},i,g}(j)(\ \llbracket\mathbf{s}\rrbracket^{\mathscr{M},i,g}))$$
$$\overset{\eta\,\text{conv.}}{=}\ \llbracket\mathbf{H}\rrbracket^{\mathscr{M},i,g}(\ \llbracket\mathbf{n}\rrbracket^{\mathscr{M},i,g},\lambda j.\ \llbracket\mathbf{S}\rrbracket^{\mathscr{M},j,g}(\lambda k.$$
$$\mathbf{A}^2\ \llbracket(\mathbf{A}^2({}^{\wedge\wedge}\mathbf{ALL})({}^{\wedge^2_0\vee^2\wedge\wedge}\mathbf{B}))(\wedge\wedge\mathbf{D})\rrbracket^{\mathscr{M},i,g}(j)(k)(\ \llbracket\mathbf{s}\rrbracket^{\mathscr{M},i,g}))$$
$$\overset{A8}{=}\ \llbracket\mathbf{H}\rrbracket^{\mathscr{M},i,g}(\ \llbracket\mathbf{n}\rrbracket^{\mathscr{M},i,g},\lambda j.\ \llbracket\mathbf{S}\rrbracket^{\mathscr{M},j,g}(\lambda k.$$
$$\llbracket\mathbf{A}^2({}^{\wedge\wedge}\mathbf{ALL})({}^{\wedge^{2}0\vee^2\wedge\wedge}\mathbf{B}))\rrbracket^{\mathscr{M},i,g}(j)(k)\ \llbracket{}^{\wedge\wedge}\mathbf{D}\rrbracket^{\mathscr{M},i,g}(j)(k)({}^{\mathscr{M},i,g}))$$
$$\overset{A8}{=}\mathbf{H}^{\mathscr{M},i,g}(\ \llbracket\mathbf{n}\rrbracket^{\mathscr{M},i,g},\lambda j.\ \llbracket\mathbf{S}\rrbracket^{\mathscr{M},j,g}(\lambda k.$$
$$\llbracket{}^{\wedge\wedge}\mathbf{ALL})\rrbracket^{\mathscr{M},i,g}(j)(k)(\ \llbracket{}^{\wedge^2_0\vee^2\wedge\wedge}\mathbf{B}\rrbracket^{\mathscr{M},i,g}(j)(k))\ \llbracket{}^{\wedge\wedge}\mathbf{D}\rrbracket^{\mathscr{M},i,g}(j)(k)(\ \llbracket\mathbf{s}\rrbracket^{\mathscr{M},i,g}))$$
$$\overset{A6}{=}\ \llbracket\mathbf{H}\rrbracket^{\mathscr{M},i,g}(\ \llbracket\mathbf{n}\rrbracket^{\mathscr{M},i,g}\lambda j.\ \llbracket\mathbf{S}\rrbracket^{\mathscr{M},j,g}(\lambda k.$$
$$\llbracket{}^{\wedge\wedge}\mathbf{ALL})\rrbracket^{\mathscr{M},i,g}(j)(k)\ \llbracket{}^{\wedge^2_0}\mathbf{B}\rrbracket^{\mathscr{M},i,g}(j)(k))\ \llbracket{}^{\wedge\wedge}\mathbf{D}\rrbracket^{\mathscr{M},i,g}(j)(k)(\mathbf{s}^{\mathscr{M},i,g}))$$
$$\overset{\text{not}}{=} H_i\,(\mathrm{n},\ \lambda j.\,S_j\,(s,\lambda k.\,B1)\subseteq(D_k))$$

$$\llbracket|36|^0_1\rrbracket^{\mathscr{M},i,g}$$
$$= \ldots$$
$$= H_i\,(n,\lambda j.\,S_j\,(s,\lambda k.\,B_j\subseteq D_k))$$

$$\llbracket|36|^0_2\rrbracket^{\mathscr{M},i,g}$$
$$= \ldots$$
$$= H_i\,(n,\lambda j.\,S_j\,(s,\lambda k.\,B_k\subseteq D_k))$$
$$\llbracket\,|(36)|\rrbracket^{\mathscr{M},i,g}$$

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Notes

1 The gappy rest will be taken to denote a predicate obtained by abstraction, for example, by a specific variable-binding mechanism or by functional composition. Cf. Szabolcsi (2011) for a survey of the options (and much more).

2 If the language under scrutiny contains variable-binding operations, intensions do not suffice as compositional values; cf. (Zimmermann and Sternefeld, 2013, ch. 10). This complication will be ignored throughout this text. Also, context dependence and token reflexivity, both of which play a central rôle in Montague's (1970) theory of reference, will be suppressed in this chapter. Depending on whether the infamous ban on monsters from (Kaplan, 1989, 510ff.) can be upheld, this may turn out to be a simplification.

3 The English fragments in Montague (1970, 1973) are famous exceptions, subsuming predication under more general patterns. It should be noted that (3) does not characterize a total function on all intensions. Rather, the current setting diverges from Montague's (1970) framework by taking syntactic and semantic operations to be (possibly) partial functions; see Janssen (1983: 421ff.) for the relevant algebraic background.

4 Cf. Montague (1970, 1973). Of course, this is not the only analysis of the construction; see Forbes (2013) and Schwarz (Forthcoming) for surveys of its alternatives.

5 The appropriateness requirement in (9) is meant to ensure that only expressions are considered that can undergo the syntactic operation $\mathscr{F}$. In (10) appropriateness is understood so as to guarantee that, for any k between 1 and n, x_k and x'_k are intensions of the same type (viz., ($\boldsymbol{s}$, τ_κ), where κ is the category of Δ_k).

6 Cf. Zimmermann (2011: 768), where these extrapolations are said to reflect 'a certain amount of idealisation'.

7 Of course, binarity is not essential; see Zimmermann and Sternefeld (2013: 197f., fn. 27) for a unary operation with the same properties. What is crucial, though, is the *syncategorematicity* of $\mathscr{F}^{\#}$, since the extensionality criteria under scrutiny only concern grammatical *constructions*, not (functional) *expressions*, which combine by functional application. I am indebted to Hannes Leitgeb for discussion of this point.

8 More specifically, a uniform operation $\|\mathscr{F}\|_*$ would have contradictory properties, as can be seen by considering some point $i \neq i^{\#}$ and the characteristic functions $\top$ and $\bot$ of $D\boldsymbol{s}$ and $\emptyset$, respectively:

$$\Big\|\mathscr{F}\Big\|(\top,\bot)(i^{\#}) \overset{(12)}{=} \Big\|\mathscr{F}\Big\|_*(\top(i^{\#}),\bot\,(i^{\#})) \overset{\text{def }\top,\bot}{=} \Big\|\mathscr{F}\Big\|_*(1,0) \overset{\text{def }\top,\bot}{=}$$
$$\Big\|\mathscr{F}\Big\|_*(\top(i),\bot\,(i)) \overset{(12)}{=} \Big\|\mathscr{F}\Big\|(\top,\bot)(i)$$

Yet given (13):

$$\|\mathscr{F}\|(\top,\bot)(i^{\#}) = max(\top(i^{\#}),\bot\,(i^{\#})) = 1,$$

whereas

$$\|\mathscr{F}\|(\top,\bot)(i) = min(\top(i),\bot\,(i)) = 0.$$

9 See, for example, van Benthem (1995) or Zimmermann (2011).

10 *Ty*2, some familiarity with which is assumed here, is a straightforward two-sorted version of the simple theory of types as defined in Church (1940); the notation used here follows standard semantic applications. Appendix B makes use of Montague's (1970) intensional type logic *IL* instead, which may be construed as a sub-language of *Ty*2; see Gallin (1975: 61ff.) and Zimmermann (1989) for more on the relation between the two languages. The following *Ty*2-formula can be used to define $\|\mathscr{F}^{\#\#}\|$:

$$\boldsymbol{\lambda\varphi.\,\lambda\psi.\,\lambda i.\,\lambda j.}$$
$$[[[\boldsymbol{\varphi(i)(i)} \wedge \boldsymbol{\psi(i)(i)}] \wedge [\boldsymbol{\varphi(i)(j)} \wedge \boldsymbol{\psi(i)(j)}]] \vee [[\neg\boldsymbol{\varphi(i)(i)} \vee \neg\boldsymbol{\psi(i)(i)}] \wedge [\boldsymbol{\varphi(i)(j)} \vee \boldsymbol{\psi(i)(j)}]]]$$

To show the non-uniformity of $\|\mathscr{F}^{\#\#}\|$, one may apply it to (the characteristic functions of) a singleton {*i*} and the set *D**s*** at *i* and *j* (≠ *i*) and compare the results in the style of fn. 8 above.
It should be noted that an invariant construction $\|\mathscr{F}\|$ can only be uniformly extensional if the type of at least one of its arguments contains an ***s***. If not, any permutation π that maps all individuals to themselves would leave the values of $\|\mathscr{F}\|$ untouched so that the construction, if invariant, would have to be constant across Logical Space—formally:

$$\pi(\|\mathscr{F}\|)(x_1,\ldots,x_n)(i) = \pi(\|\mathscr{F}\|(\pi(x_1),\ldots,\pi(x_n)))(\pi(i)) = \|\mathscr{F}\|(x_1,\ldots,x_n)(\pi(i)).$$

11 Groenendijk and Stokhof (1982). Objects of type (***s***, (***s***, ***t***)) will also play a systematic rôle (as indirect intensions) in Sections 10.4 and 10.5.

12 Cf. Zimmermann (1991: 167).

13 It is worth noticing that, like Carnapian intensions, Fregean senses behave compositionally in (uniformly) extensional constructions, due to the doctrine that sense determines reference: 'a thesis […] that is frequently attributed to Frege, although never explicitly endorsed by him' (Textor, 2011, 33).

14 In fact, grammatical meaning largely boils down to functional application in this framework; cf. Heim and Kratzer (1998: 13).

15 By putting $\|\mathscr{F}\|_i(x,y) \overset{\text{def}}{=} \|\mathscr{F}\|(x,y)(i)$, for any (suitable) *x* and *y*.

16 Given that, in (23), the extensions of attitude verbs are supposed to be relations S between individuals x and propositions *p*, the equation (24) boils down to the following generalized truth condition:

$$(*)\ \|\mathscr{A}\|_i(S,\,p)(x) = 1 \text{ iff } \mathrm{S}\,(p)\,(\mathrm{x}) = 1.$$

Alternatively, following Hintikka (1969), one may take attitude verbs to denote relations A between individuals and points in Logical Space (the pertinent alternatives), in which case one would get:

> (+) $\|\mathscr{A}\|_i(A,\, p)(\mathrm{x}) = 1 \text{ iff } p(j) = 1, \text{whenever } \mathrm{A}(j)(\mathrm{x}) = 1.$

A preference for (+) over the less specific condition (*) would not bear upon the considerations to follow.

17 Even the kind of contextual variation of semantic constructions defended by Lasersohn (2012) does not *per se* contradict the uniformity criterion (25), though it does seem to go against the spirit of Fregean compositionality.

18 At least this is how certain passages in Frege (1892) are frequently construed; see, for example, Carnap (1947: §30) or Dummett (1973: 266f.). However, as pointed out by Parsons (1981), this 'orthodox interpretation' finds no obvious support by textual evidence, although Frege may have intended it. See also Kutschera (1974: 64).

19 This has been made popular by Heim and Kratzer (1998), if for different reasons. In most of the earlier formal semantics literature, starting with Montague (1970), *λ*-abstraction had been confined to the formal language of indirect interpretation, a notable exception being Kutschera (1974: 230).

20 It may be worth pointing out that the alphabetic variation on the *λ*-bound (meta-linguistic) variables in (27e) has only been made for readability; the last line is equivalent to:

> $H_i(\mathrm{n}, \lambda i.\, S_i(\mathrm{s}, \lambda i.\, P_i(\mathrm{e})))$

where all index variables coincide with the default variable '*i*'.

21 Superscripts are used to indicate iterated concatenation of (meta-linguistic) strings.

22 Cf. Davidson (1968: 11).

23 The connection between Bäuerle's (1983) problem and Frege's hierarchy was touched on but dismissed in (Zimmermann, 2012: 644, fn. 29).

24 In fact, if predicates were conceived of as sentences with subject gaps (as in categorial grammar), one could do without a separate subject construction $\mathscr{S}$ altogether, subsuming it under general quantification $\mathscr{Q}$.

25 Arguably, the two readings in (34) might be equivalent after all, for example, due to lexical properties of the attitude verb. To control for this and other distracting side-effects, the examples in Bäuerle (1983) are actually somewhat more involved.
Since the publication of Bäuerle (1983), similar cases have been discovered, discussed and analysed in different ways, usually involving multiple variable binding; cf. Cresswell (1990), Percus (2000), or Keshet (2010a,b). A comparison of these analyses with the current approach is beyond the scope of this chapter.

26 The solution is somewhat in the spirit of what (Bäuerle, 1983, 128ff.) indicates under the heading *Opazität ohne Skopus* ['Opacity without scope'].

27 A more detailed derivation with references to the pertinent clauses can be found in Appendix A.

28 The underlying (syntactic) concepts of shifting and (immediate) scope could be made more precise. However, at this point it suffices to see the basic idea and motivation behind the construction. Appendix B contains a more rigorous treatment.

29 Bäuerle (1983: 127) seems to assume so, crediting (and quoting) Saarinen (1979) for the postulate that 'it ought to be possible for a semantics to refer back to any of a finite number of points of reference that have been introduced in the course of an evaluation' (translation by TEZ).

30 This is a special case of what Percus (2000: 201) calls *Generalization X*.

31 There is an additional twist to the first option: it would ultimately have to rely on an ambiguity in the indefinite determiner, along the lines of Heim and Kratzer (1998: 61f.); the ambiguity does not show in the appendix because predication is not part of the fragment. There is an additional twist to the second option too: it would ultimately have to rely on the rigidity of determiner extensions, along the lines of Heim and Kratzer (1998: 305), protecting them against sense twisting.

32 That *IL* suffices for expressing the operations sketched above follows from their (obvious) definability in *Ty*2 and Theorem III in Zimmermann (1989: 75). Baroque intensionality thus readjusts 'the limits of semantic analyses inspired by modal and tense logic', which were made responsible for the noncompositionality of Bäuerle's constellation in Zimmermann (2012: 644, fn. 28).

33 We thus identify truth values with the ordinals 0 [= ∅] and 1 [= {∅}].